ECE 36 - Signals and Systems.

Module - I

Syllabus :-

Classification of Signals : Continuous time signals, Discrete time signals, Periodic and Aperiodic signals, Even and Odd signals, Energy and Power signals, Deterministic and random signals.

Elementary Signal / Functions :- Complex exponential, Sinusoidal signals, Unit step, Unit ramp, Unit Impulse.

Basic operation on signals :- Amplitude Scaling, addition, multiplication, time scaling, time shift and time reversal.

Signal :- A signal can be defined as a function that conveys information. Signals are represented mathematically as a function of one or more independent variables. → If the function depends on a single variable, the signal is said to be One dimensional. eg:- Speech signal. A speech signal is represented mathematically as a function of time where, the amplitude varies with time depending on the spoken word and the person who speaks it.

→ If the function depends on two or more variables, the signal is said to be multidimensional.

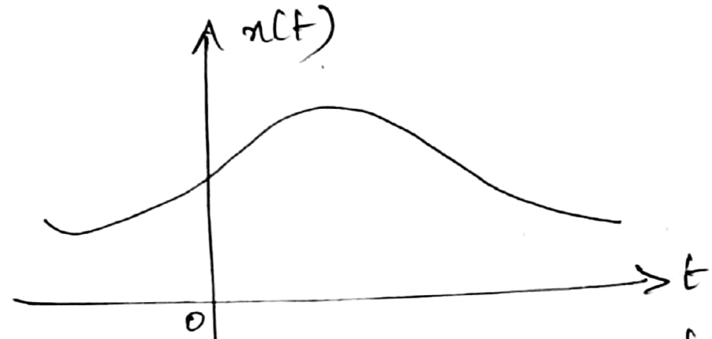
eg: Image signal. An image is a brightness function of two spatial variables.

Classification of Signals.

(i) Continuous-time and Discrete-time Signals.
(ii) Even and odd Signals.
(iii) Periodic and Non-Periodic Signals.
(iv) Deterministic and Random Signal.
(v) Energy and Power Signal.

(i) → Continuous-time Signal.

A signal $x(t)$ is said to be a Continuous-time Signal if it has value of amplitude for all time 't'.



Above figure represents an example of a Continuous-time Signal whose amplitude varies continuously with time. A Continuous-time Signal arise naturally when a physical phenomenon (eg: heart beat, pressure variation etc) is converted to an electrical signal using appropriate transducer.

→ Discrete-time Signal.

* A discrete-time Signal is defined only at discrete instants of time, ie, Independent variable has discrete values only which are uniformly spaced.

* A discrete-time Signals are represented mathematically as Sequence of numbers. A sequence of numbers 'x' in which the n^{th} number in the Sequence

is denoted by $x(n)$

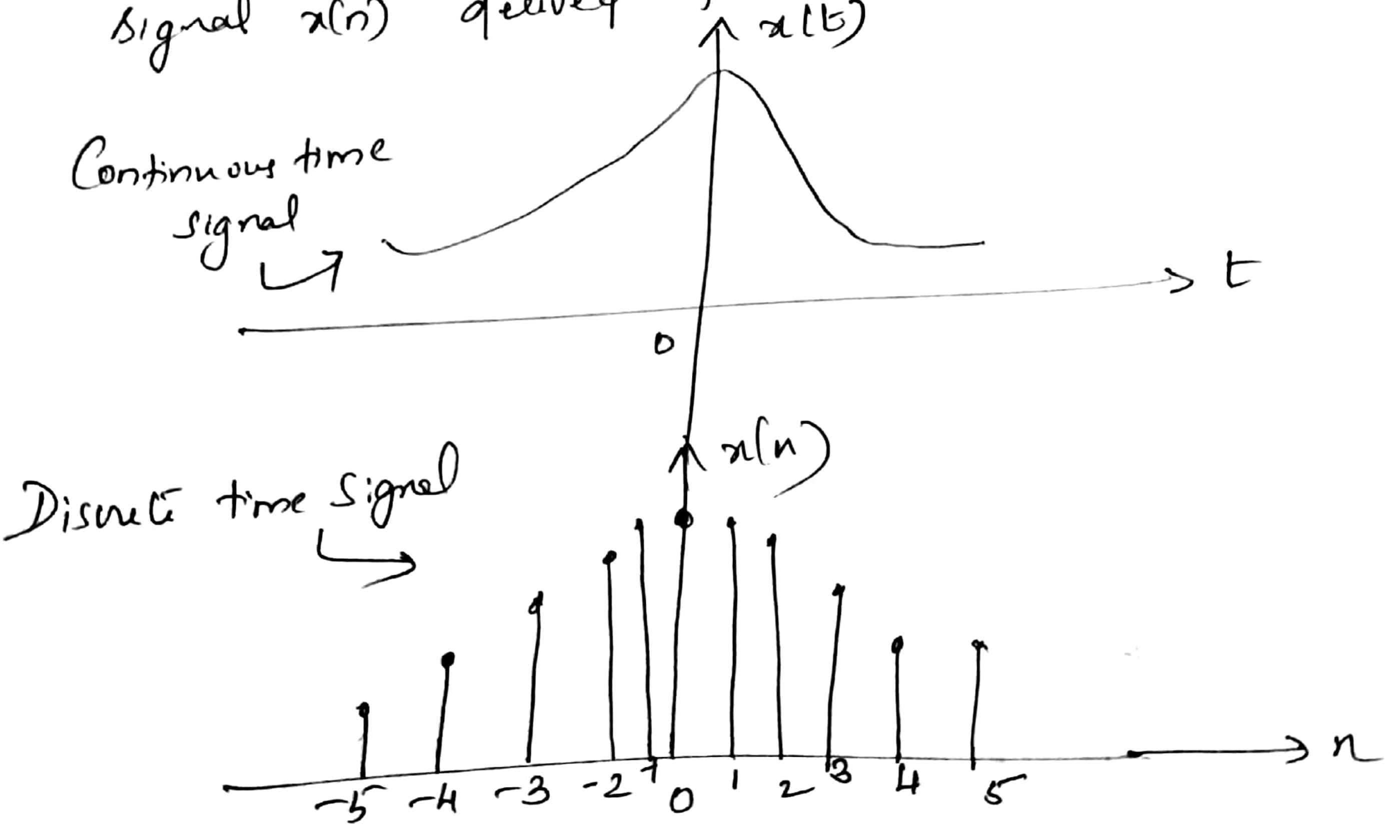
$x = \{x(n)\}$; $-\infty < n < \infty$.

where 'n' is an Integer.

* Consider 'τ' is the Sampling Period and 'n' denote an Integer $(-\infty < n < \infty)$. Sampling a Continuous-time Signal $x(t)$ at time $\underline{t = n\tau}$ gives a sample value $x(n\tau)$. Thus, sampled Signal $x(n)$ is

$$\boxed{x(n) = x(n\tau)} \; ; \; n = 0, \pm 1, \pm 2, \pm 3 \cdots$$

* Fig below illustrates the relationship between a Continuous-time signal $x(t)$ and discrete-time signal $x(n)$ derived from it.



iii) Even and odd Signal

→ **Even signal**

* A continuous-time signal $x(t)$ is said to be an even signal, if it satisfies the condition $x(-t) = x(t)$ for all 't'

* A discrete time signal $x(n)$ is said to be an even signal, if it satisfies the condition $x(-n) = x(n)$ for all 'n'

* Even signals are symmetrical about the vertical axis or the time origin.

* A continuous-time signal $x(t)$ can be decomposed into sum of two signals, one of which is even $x_e(t)$ and other is odd $x_o(t)$.

$$x(t) = x_e(t) + x_o(t) \longrightarrow (1).$$

For $x_e(t)$ to be even

$$x_e(-t) = x_e(t) \text{ —— } (2)$$

For $x_o(t)$ to be odd

$$x_o(-t) = -x_o(t) \text{ —— } (3)$$

Sub $t = -t$ in eq (1).

$$x(-t) = x_e(-t) + x_o(-t) \text{ —— } (4)$$

Put (2) & (3) in (4).

$$x(-t) = x_e(t) + -x_o(t)$$

or, $x(-t) = x_e(t) - x_o(t)$ —— (5).

Adding ① + ⑤.

$$x(t) + x(-t) = x_e(t) + \cancel{x_o(t)} + x_e(t) - \cancel{x_o(t)}$$

$$x(t) + x(-t) = 2x_e(t)$$

$$\Rightarrow \boxed{x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]} \quad \text{— ⑥.}$$

Similarly,
for discrete signal

$$\boxed{x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]} \quad \text{— ⑦.}$$

→ Odd signal .

* A continuous-time signal is said to be an odd signal if it satisfies the condition $x(-t) = -x(t)$ for all 't'.
* A discrete signal is said to be an odd signal if it satisfies the condition $x(-n) = -x(n)$. for all 'n'
* Odd signals are antisymmetric about the vertical axis or the time origin.

① − ⑤.

$$x(t) - x(-t) = x_e(t) + x_o(t) - \left[x_e(t) - x_o(t)\right].$$

$$x(t) - x(-t) = 2x_o(t)$$

or, $$\boxed{x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]} \quad \text{— ⑧.}$$

Similarly for discrete time signal.

$$\boxed{x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]} \quad \text{--- (9)}$$

1.16

Q : (1)

Find the even and odd Components of the signal.

$$x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t).$$

Sol^n

Given:- $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$ --- (1)

$$x(-t) = \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)$$

But $\left\{ \begin{array}{l} \cos(-t) = \cos t \\ \sin(-t) = -\sin t \end{array} \right\}$.

Substitute,

$$x(-t) = \cos(t) - \sin(t) - \sin(t)\cos(t) \quad \text{--- (2)}$$

(1) + (2).

$x(t) + x(-t)$

$\Rightarrow \cos(t) + \sin(t) + \sin(t)\cos(t) +$
$\cos(t) - \sin(t) - \sin(t)\cos(t).$

$= 2\cos(t).$

$$x(t) + x(-t) = 2\cos(t)$$

$$\frac{x(t) + x(-t)}{2} = x_e(t) = \cos t$$

$\therefore$ even Part $\rightarrow \boxed{x_e(t) = \cos(t)}$

 (1) - (2)

$$x(t) - x(-t) = \cos(t) + \sin(t) + \sin(t)\cos(t) - [\cos(t) - \sin(t) - \sin(t)\cos(t)]$$

$$x(t) - x(-t) = 2\sin(t)\cos(t) + 2\sin t$$

$$\frac{x(t) - x(-t)}{2} = \sin(t)\cos(t) + \sin(t)$$

$$\boxed{x_o(t) = \sin(t)[1 + \cos(t)]} \rightsquigarrow \text{odd Part.}$$

1.17

Q:(2) Obtain the even and odd components of the signal $x(t) = (1+t^3)\cos^3(10t)$.

Sol^n

Given :- $x(t) = (1+t^3)\cos^3(10t) \rightarrow (1)$

$$x(-t) = [1 + (-t)^3]\cos^3(-10t)$$

$$x(-t) = [1 - t^3]\cos^3(10t) \rightarrow (2)$$

(1) + (2)

$$x(t) + x(-t) = (1+t^3)\cos^3(10t) + (1-t^3)\cos^3(10t)$$

$$= \cos^3(10t)[1 + t^3 + 1 - t^3]$$

$$x(t) + x(-t) = 2\cos^3(10t)$$

$$\frac{x(t) + x(-t)}{2} = \cos^3(10t)$$

$$\therefore \boxed{x_e(t) = \cos^3(10t)} \rightsquigarrow \text{even Part}$$

$① - ② = x(t) - x(-t)$

$(1+t^3)\cos^3(10t) - (1-t^3)\cos^3(10t)$

$= \cos^3(10t)\left[1+t^3-1+t^3\right]$

$\Rightarrow x(t) - x(-t) = 2t^3 \cos^3(10t)$

$x_o(t) = \frac{x(t) - x(-t)}{2} = t^3 \cos^3(10t)$

$\therefore \boxed{x_o(t) = t^3 \cos^3(10t)} \rightsquigarrow$ odd Part.

1.18

$\boxed{Q:③}$ Find the even and odd Part of the signal

$x(t) = 1+t+3t^2+5t^3+9t^4$.

Sol:

Given:- $x(t) = 1+t+3t^2+5t^3+9t^4 \rightarrow ①$

$x(-t) = 1-t+3t^2-5t^3+9t^4 \rightarrow ②$

$① + ② = 1+t+3t^2+5t^3+9t^4 +$
$1-t+3t^2-5t^3+9t^4$

$x(t)+x(-t) = 2+6t^2+18t^4$

$x(t)+x(-t) = 2(1+3t^2+9t^4)$

$\frac{x(t)+x(-t)}{2} = 1+3t^2+9t^4$

$\boxed{x_e(t) = 1+3t^2+9t^4} \rightsquigarrow$ even Part

① − ②

$$x(t) - x(-t) = 1+t+3t^2+5t^3+9t^4 - [1-t+3t^2-5t^3+9t^4]$$

$$= 2t + 10t^3$$

$$x(t)-x(-t) = 2t(1+5t^2)$$

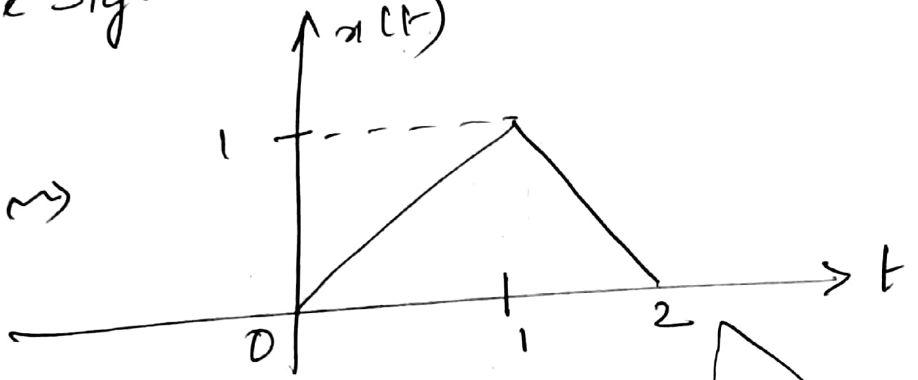
$$\frac{x(t)-x(-t)}{2} = t(1+5t^2)$$

$$\boxed{x_o(t) = t(1+5t^2)}$$ → odd Part.

1.19

Q: 4, Determine & sketch the even & odd Part of the Signal shown in figure :-

fig ① →



Sol^n Even Signal $x_e(t) = \frac{x(t) + x(-t)}{2}$

To draw $x(-t)$

fig ② →

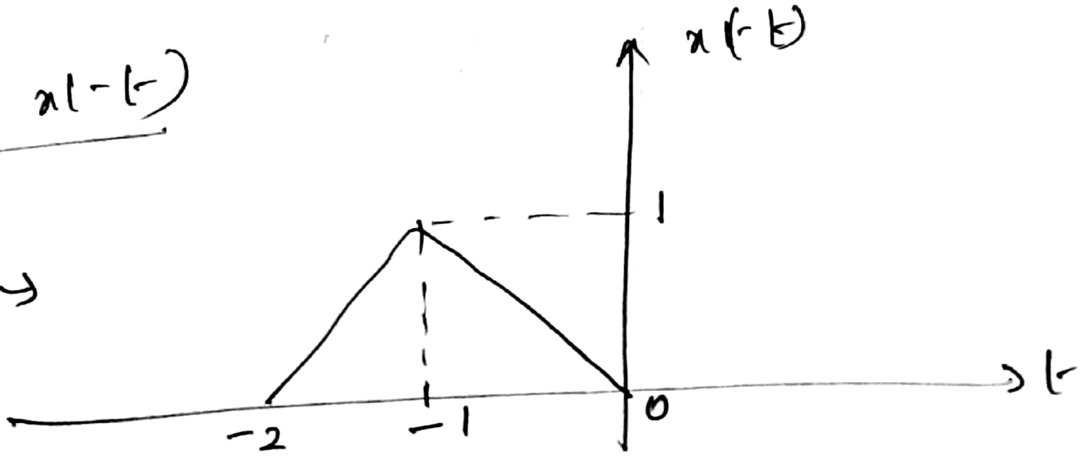
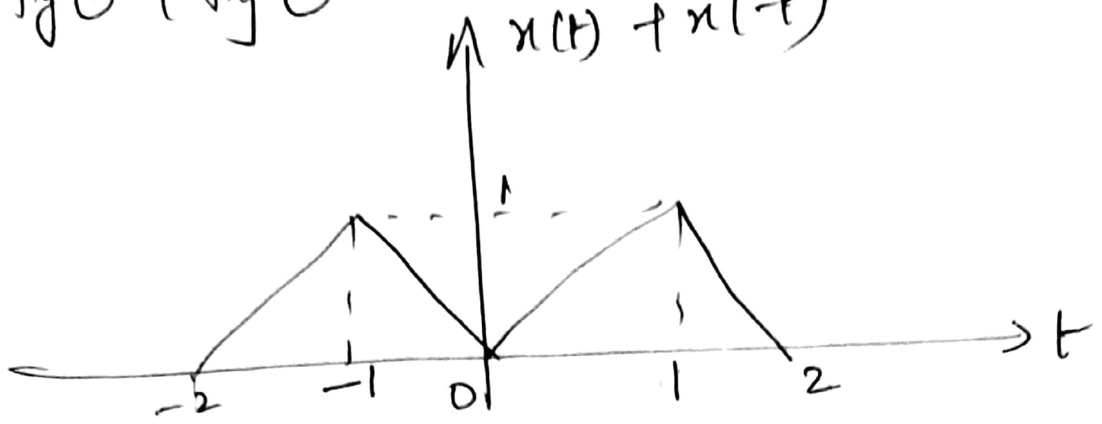
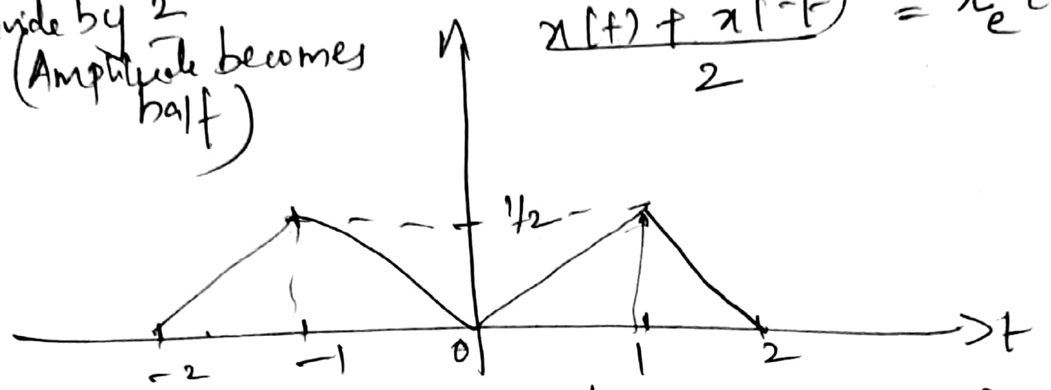


fig ① + fig ②



Divide by 2
⇒ (Amplitude becomes half)

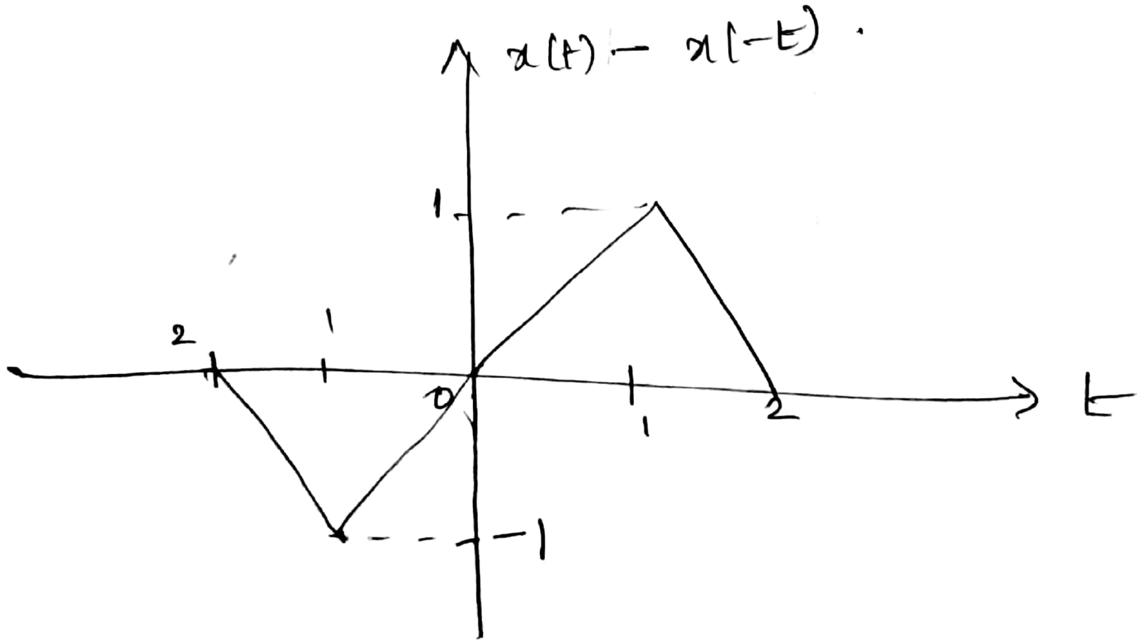
$\frac{x(t) + x(-t)}{2} = x_e(t)$.



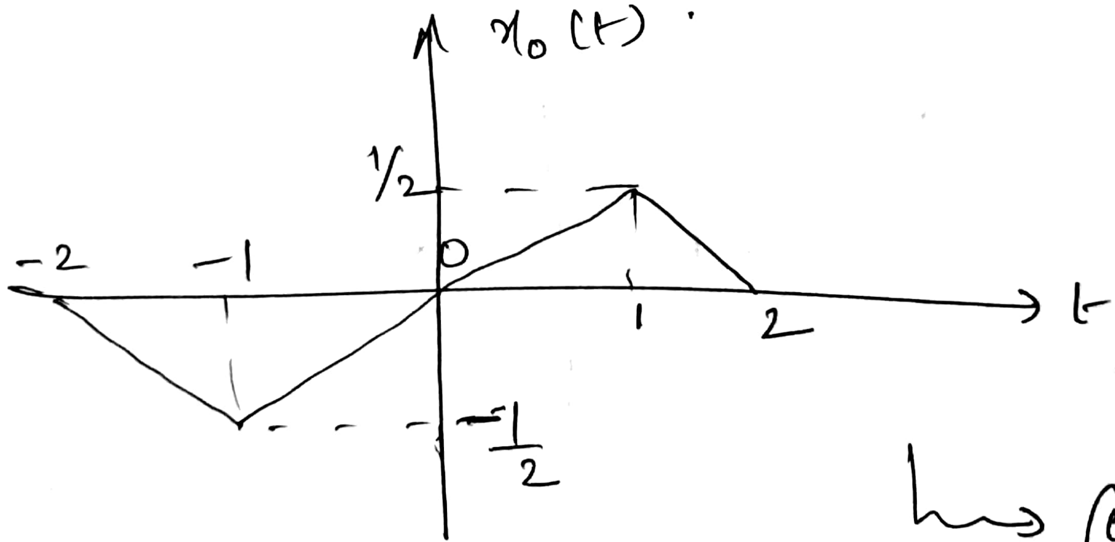
→ (Even Part)

<u>Odd Signal</u>

fig ① − fig ②



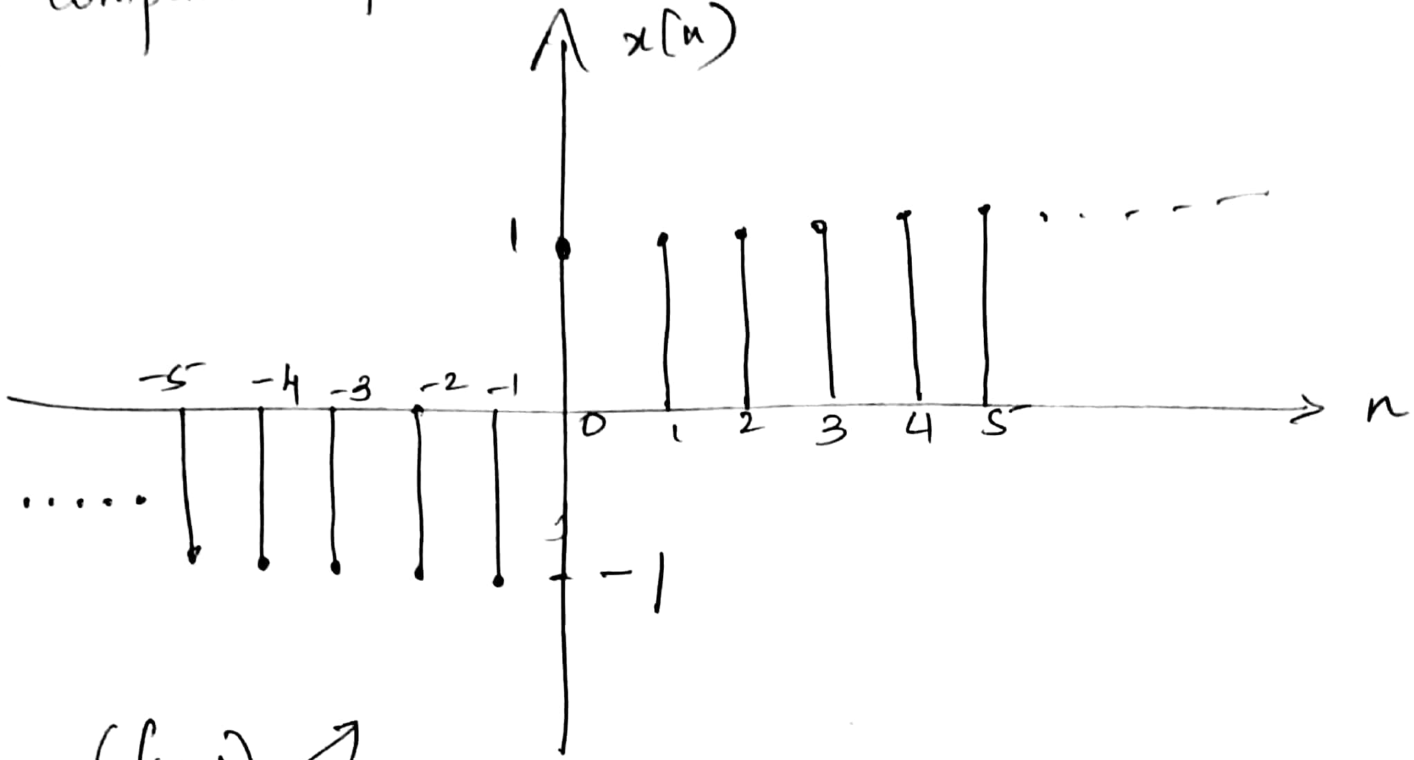
To draw $\frac{x(t) - x(-t)}{2}$; reduce the amplitude to half.



↳ (Odd - Part)

1.21

Q(6) . Determine & sketch the even and odd components of discrete time signal $x(n)$.



(fig 1)

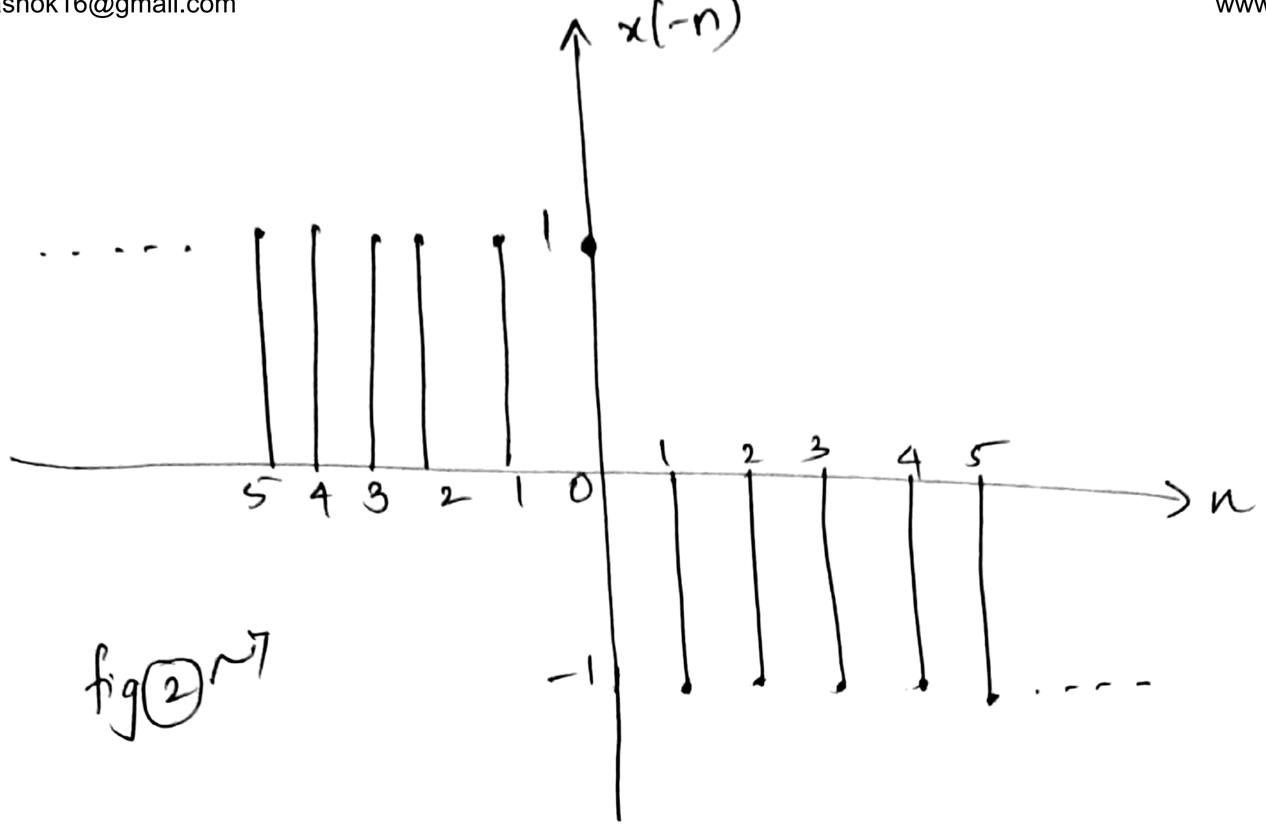
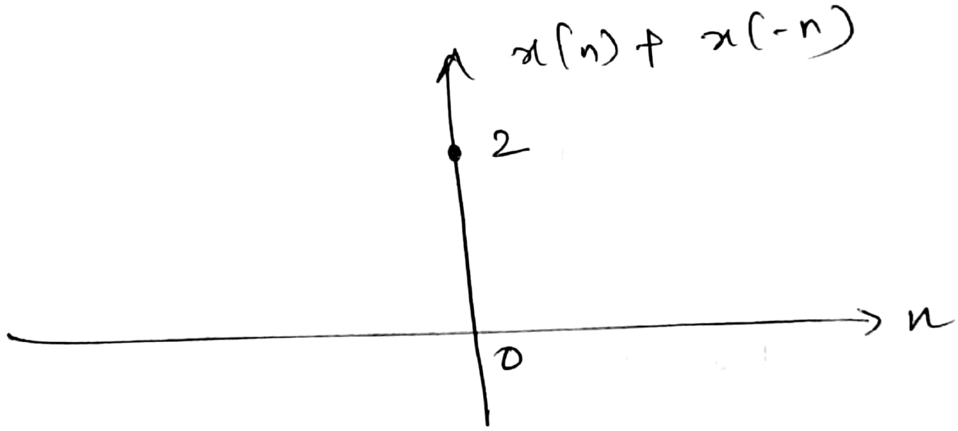


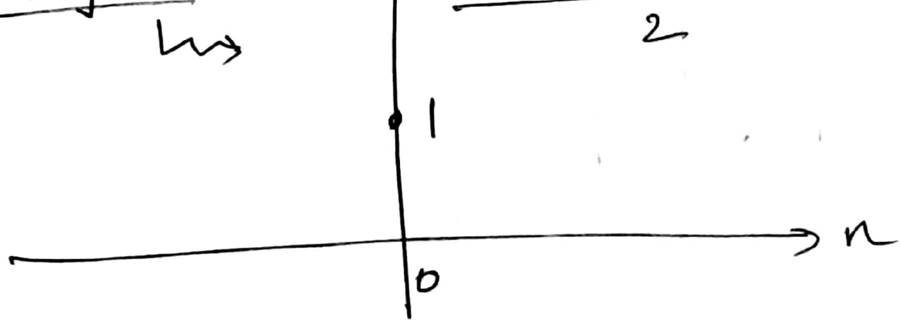
fig 2

$$x(n) + x(-n) \Rightarrow \text{fig } 1 + \text{fig } 2$$



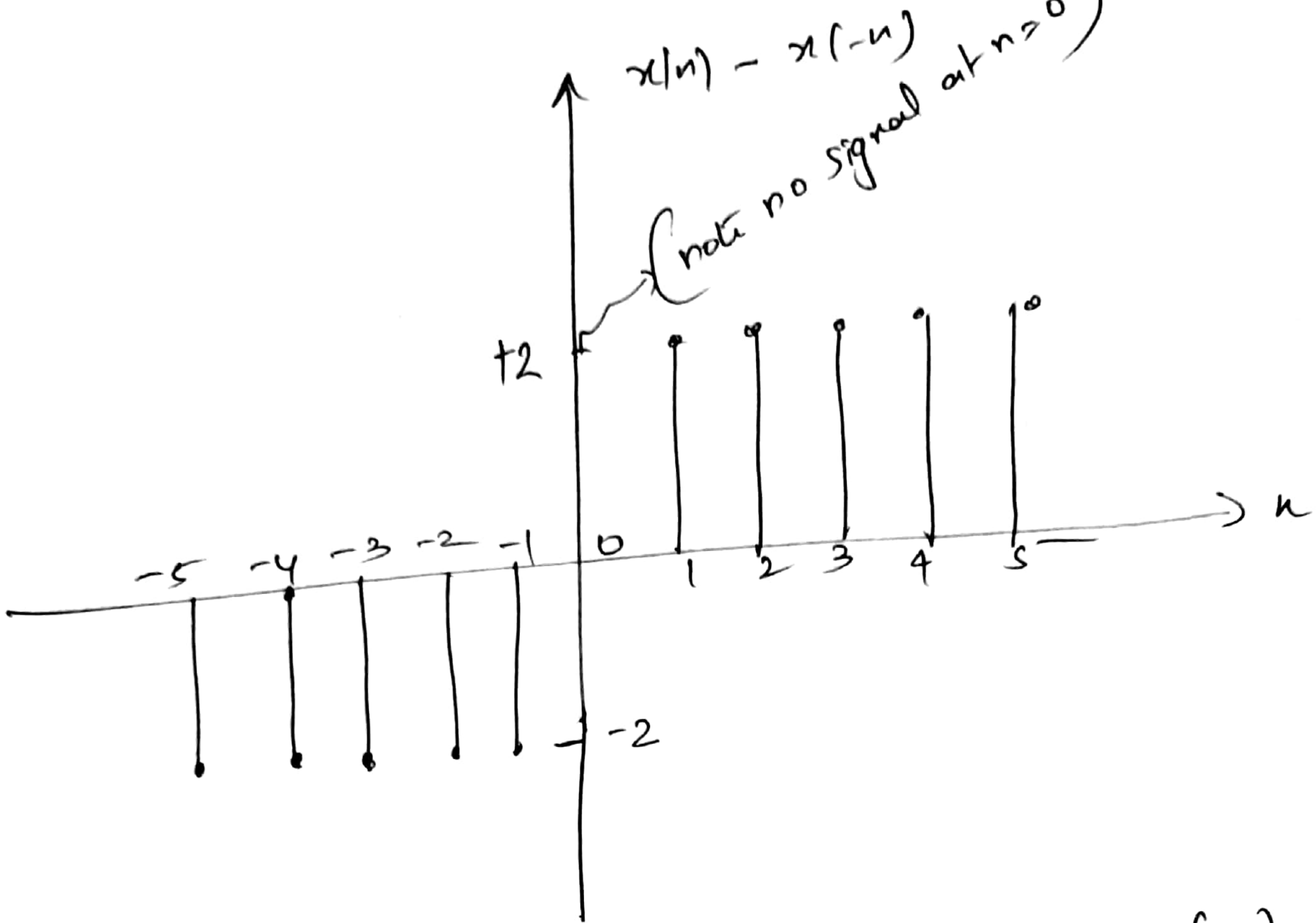
Even Signal

$$\frac{x(n) + x(-n)}{2} = x_e(n)$$

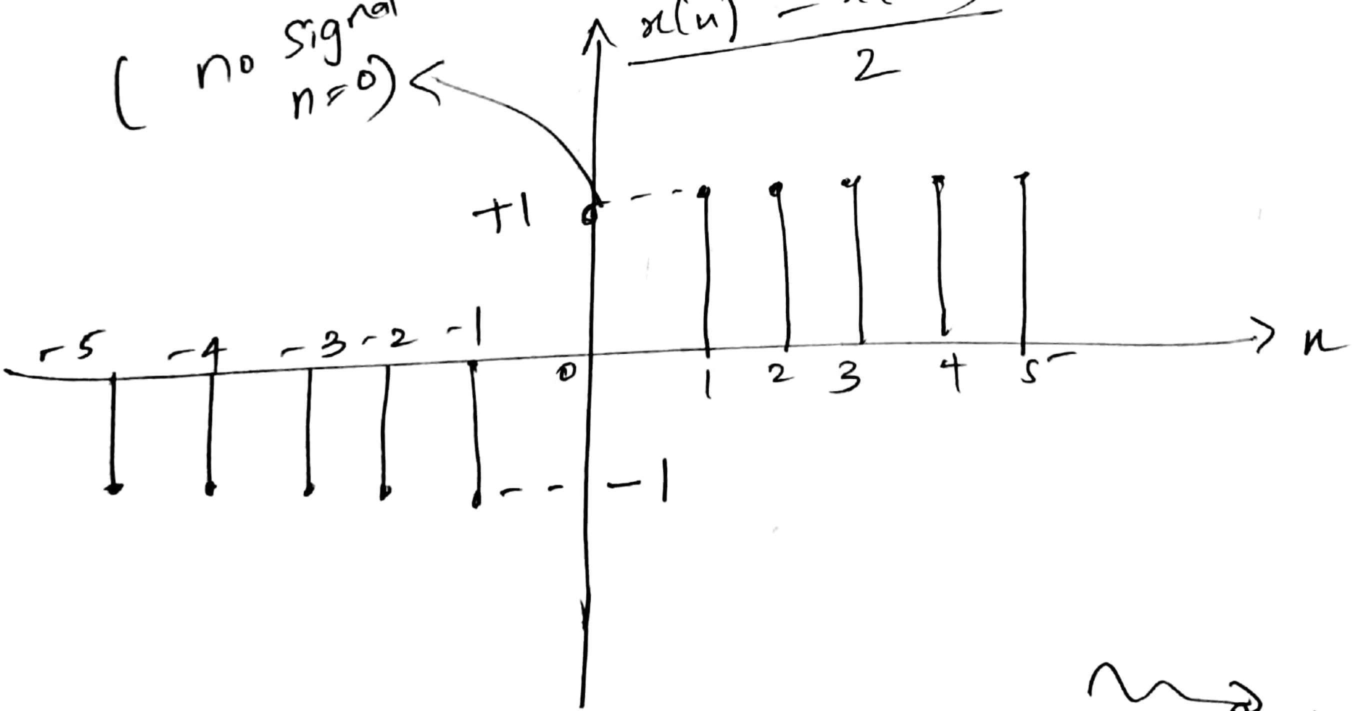


Odd Part

$x(n) - x(-n) \Rightarrow$ fig ① - fig ②.



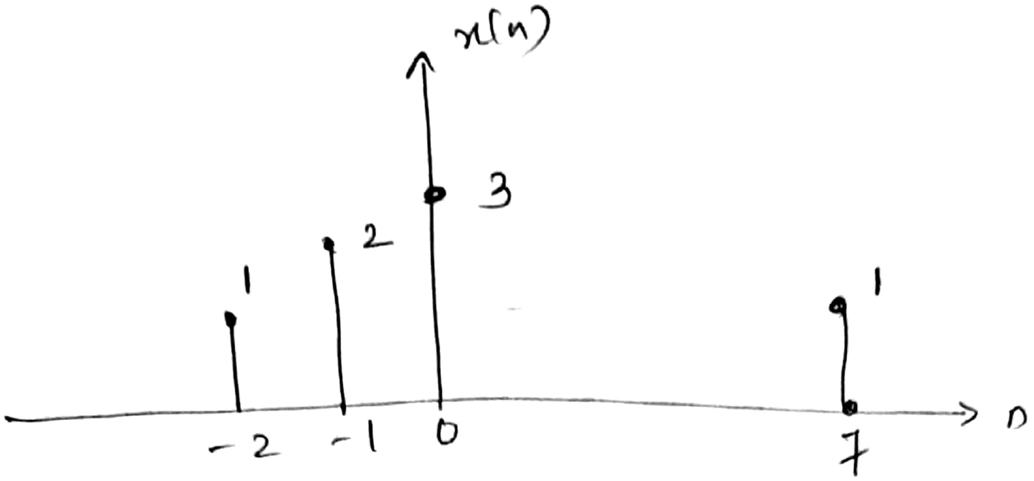
$$\frac{x(n) - x(-n)}{2} = x_0(n)$$



(Odd).

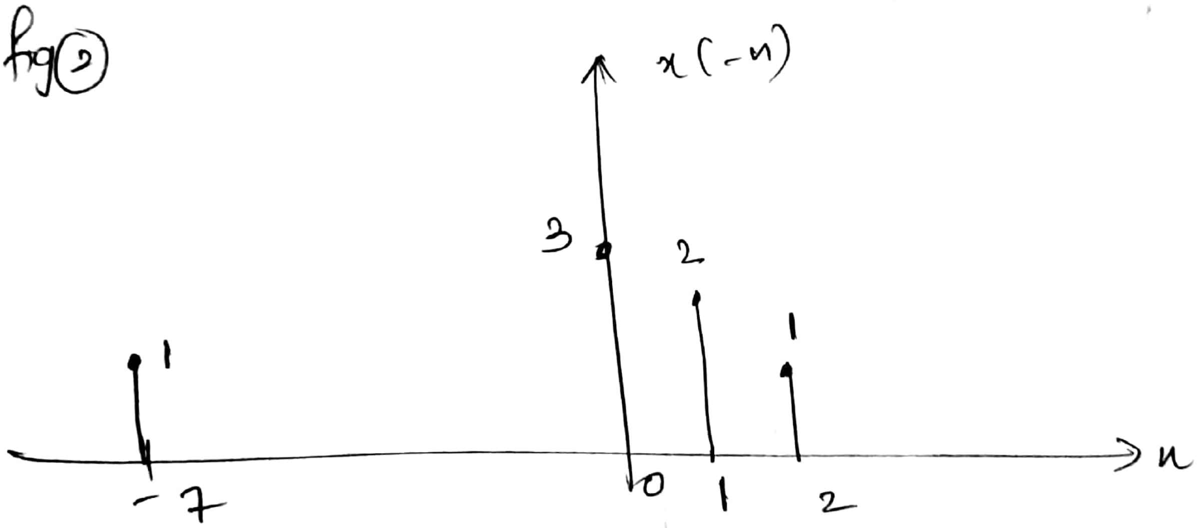
Q:7 Determine and draw the even and odd Parts of the discrete - time signal x(n) shown in figure.

fig 1



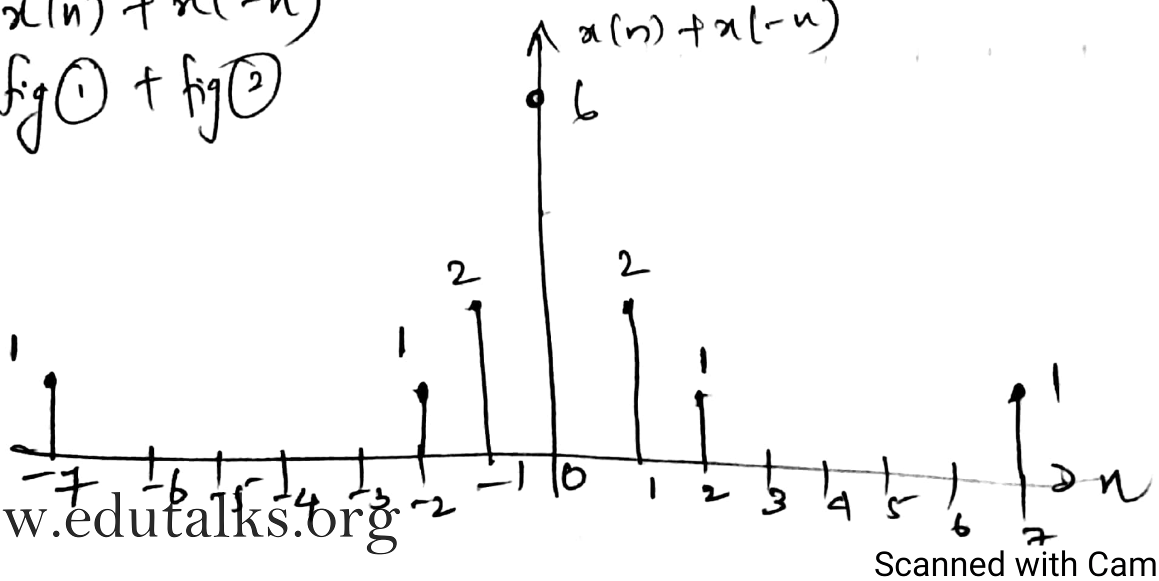
Soln

fig 2

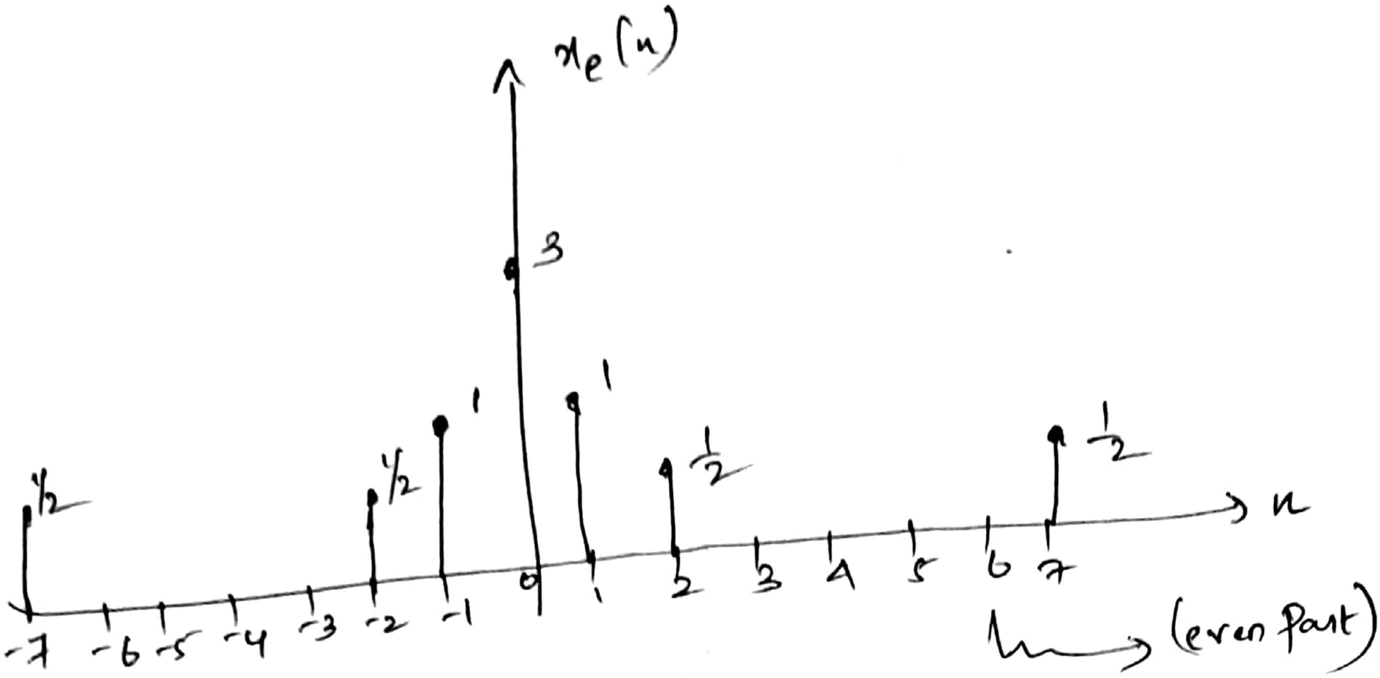


x(n) + x(-n)

fig 1 + fig 2

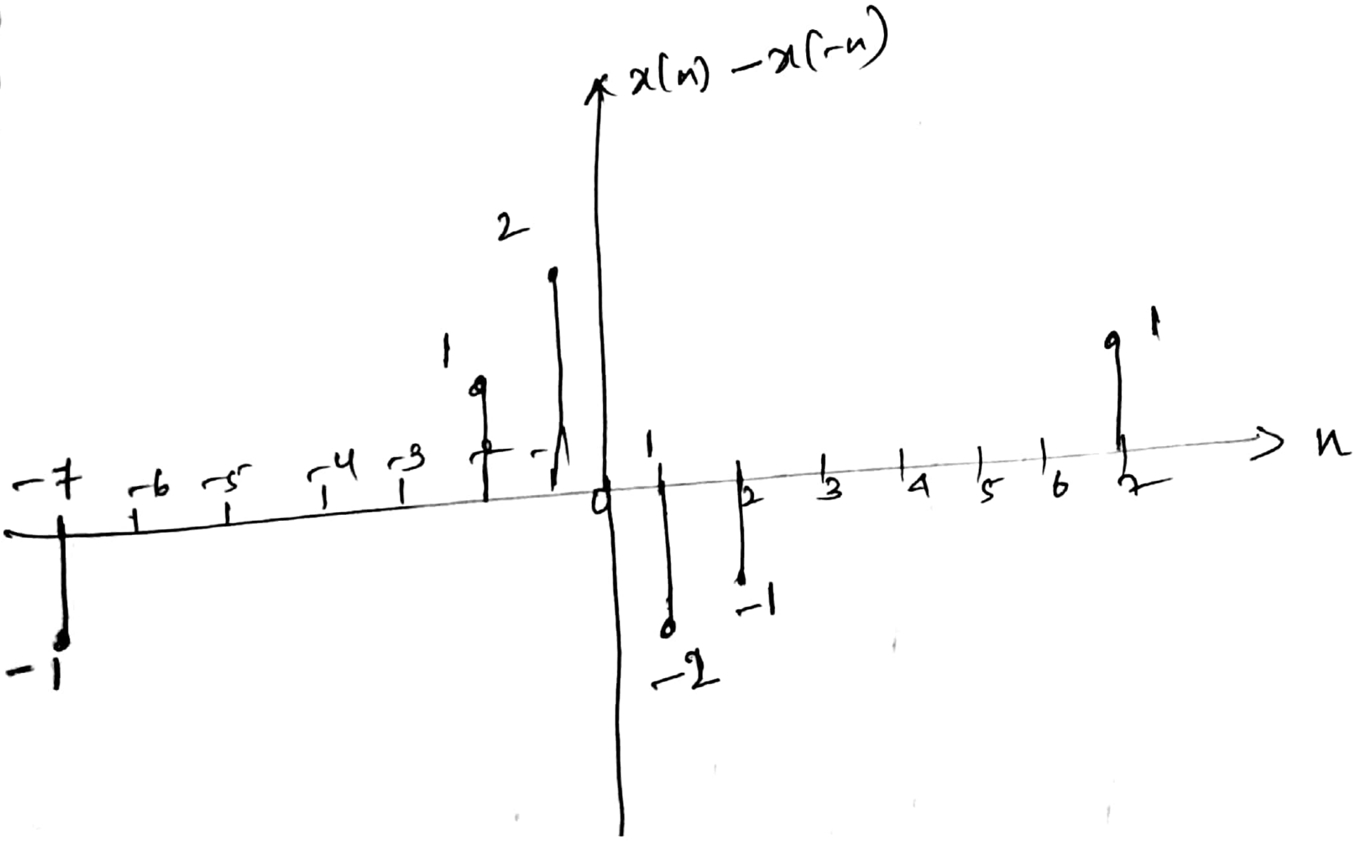


$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

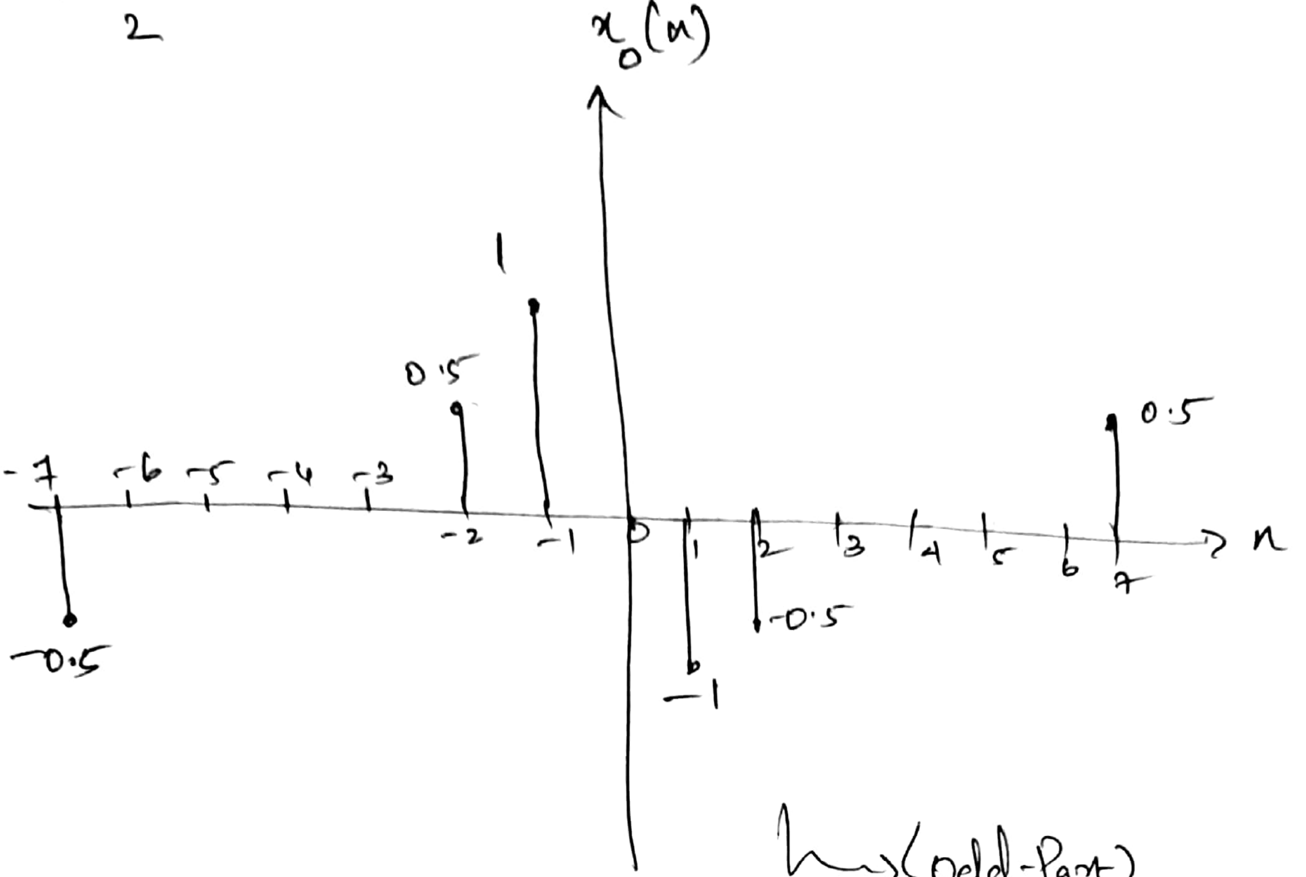


To find odd Component

$x(n) - x(-n) \Rightarrow$ fig 1 - fig 2



$\frac{x(n) - x(-n)}{2}$



→ (Odd-Part)

1.23

$\boxed{Q:8}$

find the even and odd Part of the Signal x(t) shown below :-

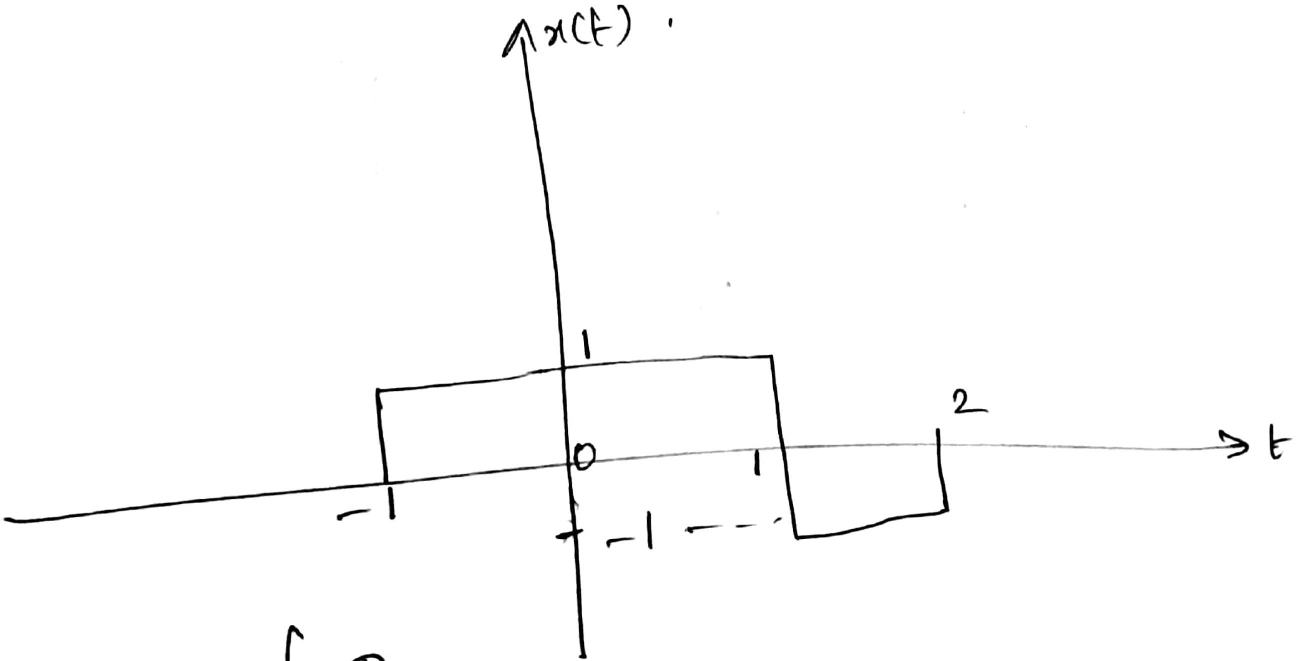
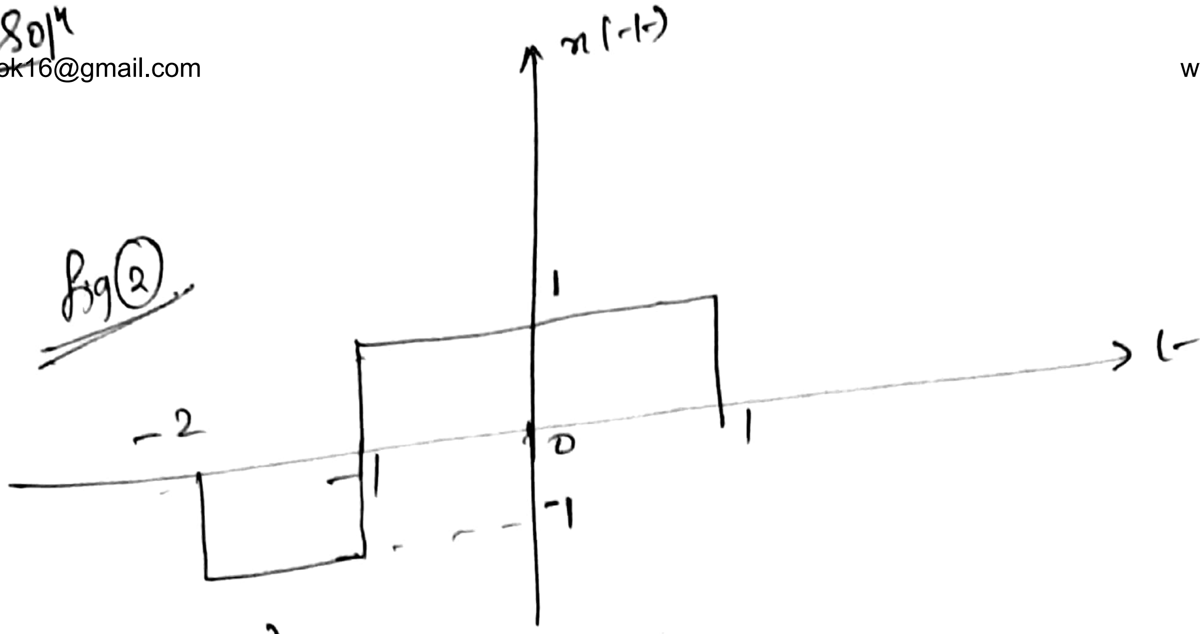
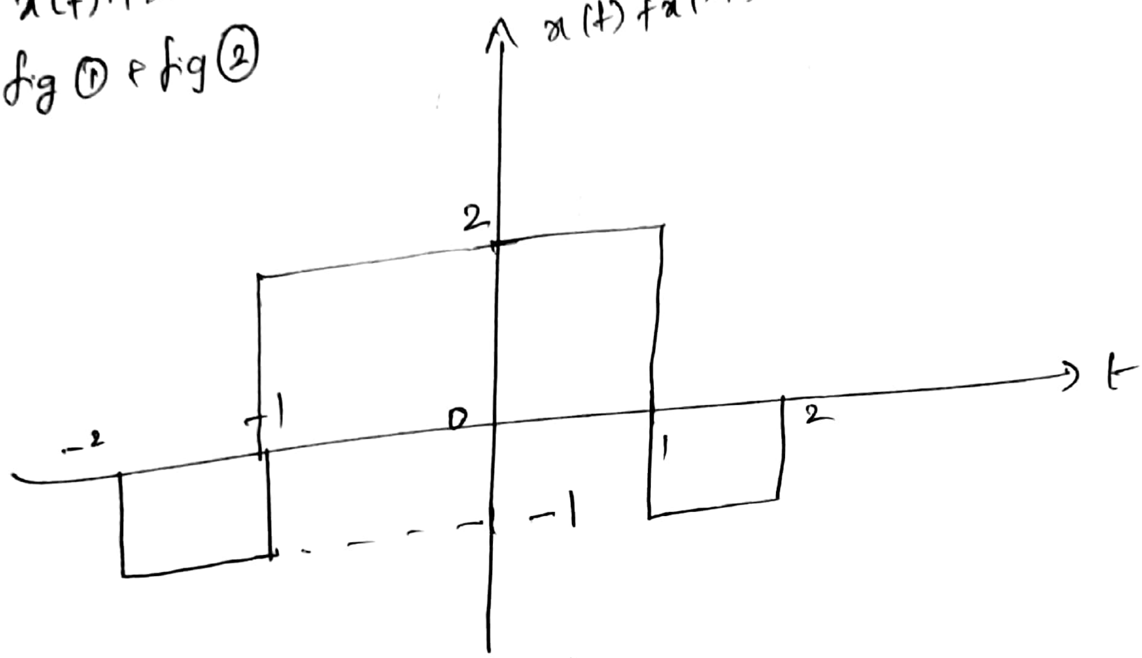


fig 1

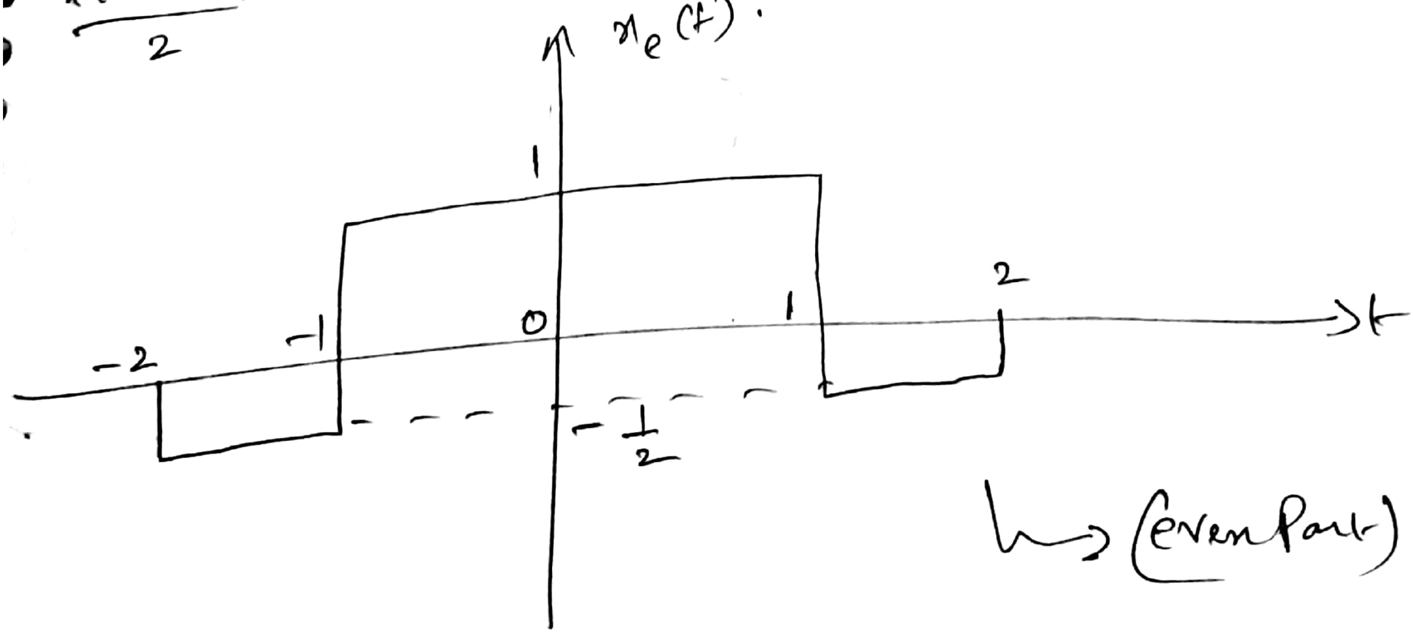
fig ②



$x(t) + x(-t)$
fig ① + fig ②



$\frac{x(t) + x(-t)}{2} = x_e(t)$

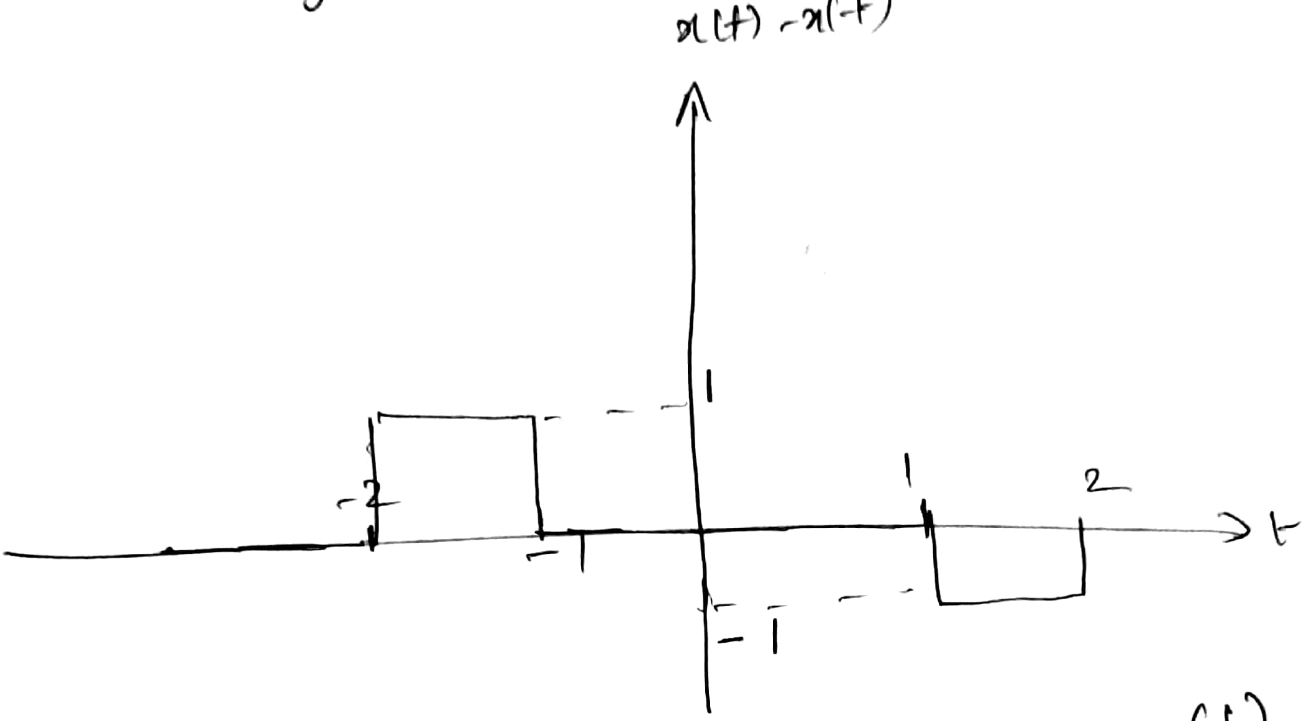


→ (even Part)

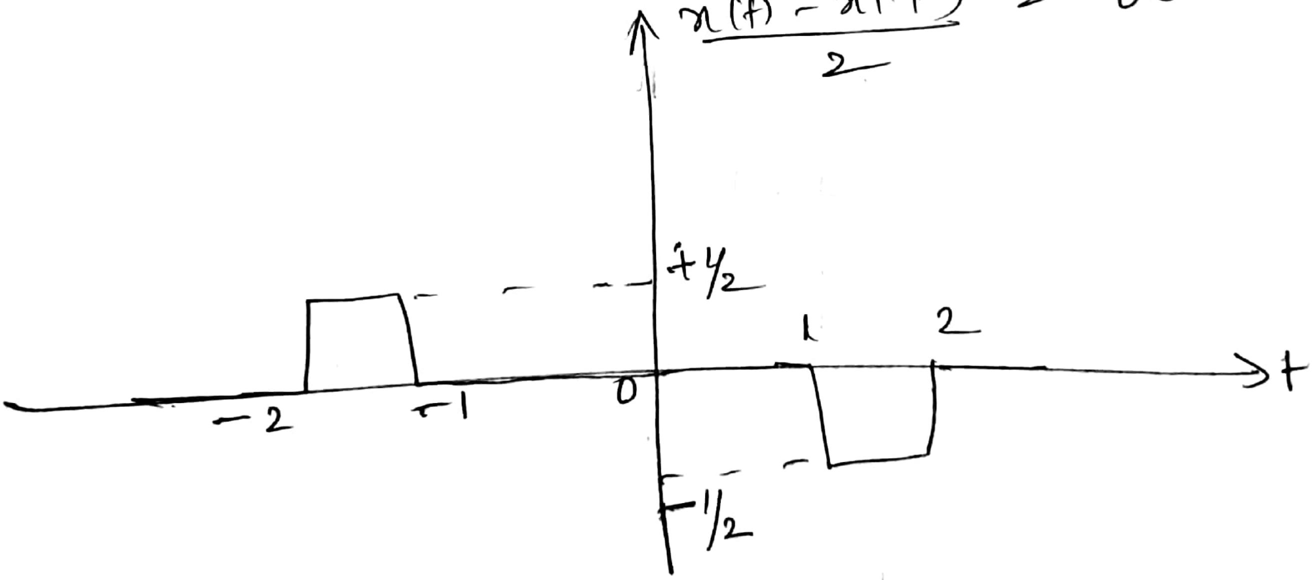
To find odd Component,

$x(t) - x(-t)$

fig ① - fig ②



$$\frac{x(t) - x(-t)}{2} = x_o(t).$$



(odd Part)

(ii) Periodic and Non-Periodic Signals.

A Continuous-time signal $x(t)$ is said to be Periodic if it satisfies the condition

$$\boxed{x(t) = x(t+T)} \text{; for all } t$$

$T \rightarrow$ fundamental time Period. (a Positive Constant)

The smallest value of $|T|$ that satisfies above equation is called fundamental Period of $x(t)$. This fundamental Period is the time taken by the signal $x(t)$ to complete its own cycle. The Reciprocal of the fundamental Period $|T|$ is known as the fundamental frequency of the signal. (f)

$$f = \frac{1}{T} \text{ (in Hertz)}$$

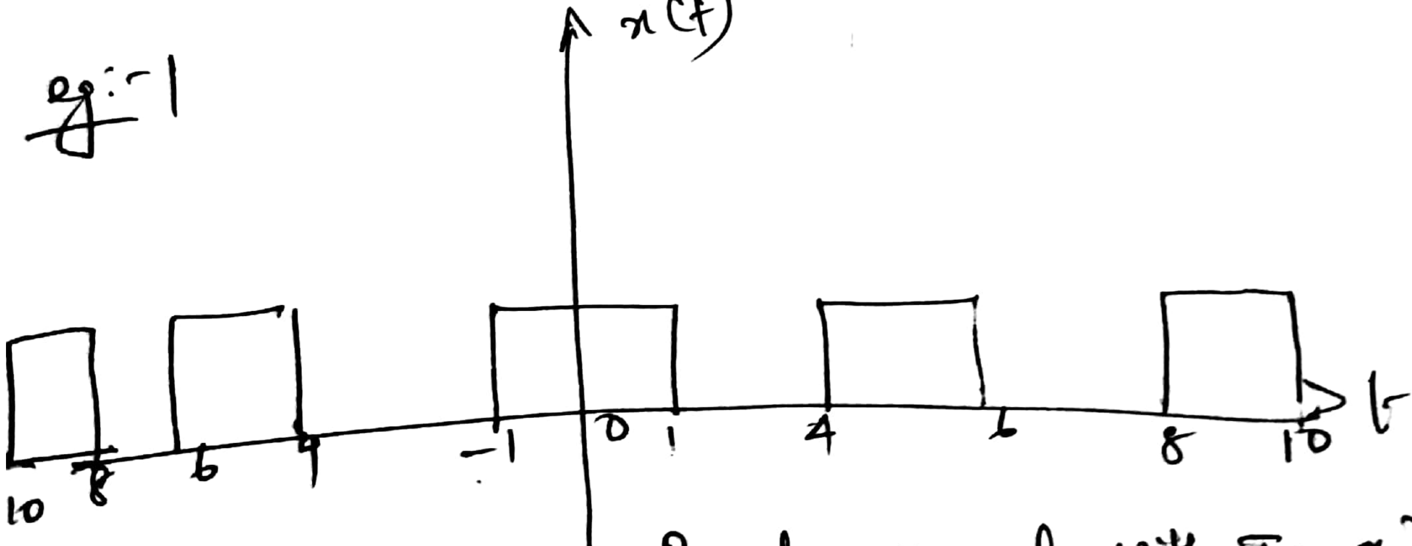
$T \rightarrow$ Sec

Fundamental angular frequency 'ω' is given by

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{(rad/sec)}$$

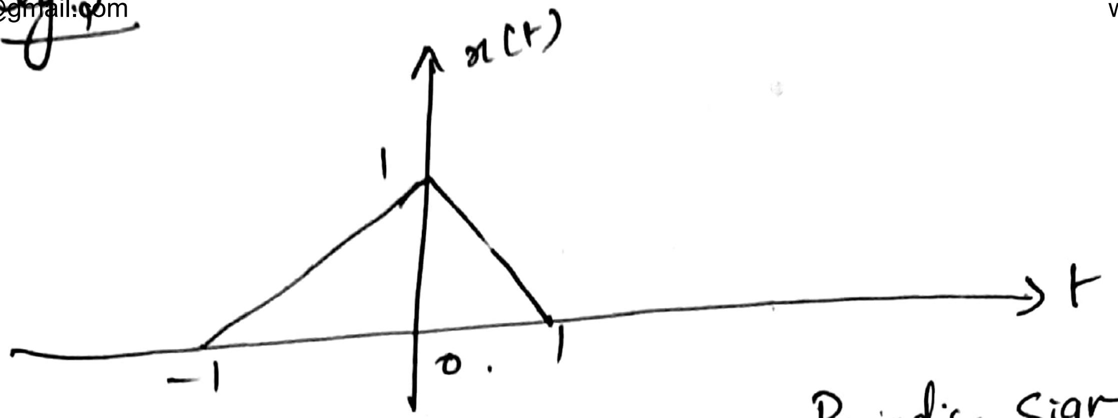
If $x(t) \neq x(t+T)$ then the signal is called non-periodic signal.

eg:-1



(Continuous-time Periodic signal with $T = 4$)

eg:



(eg: A Continuous time non-Periodic Signal)

Similarly, a discrete time signal $x(n)$ is said to be Periodic if it satisfies the condition,

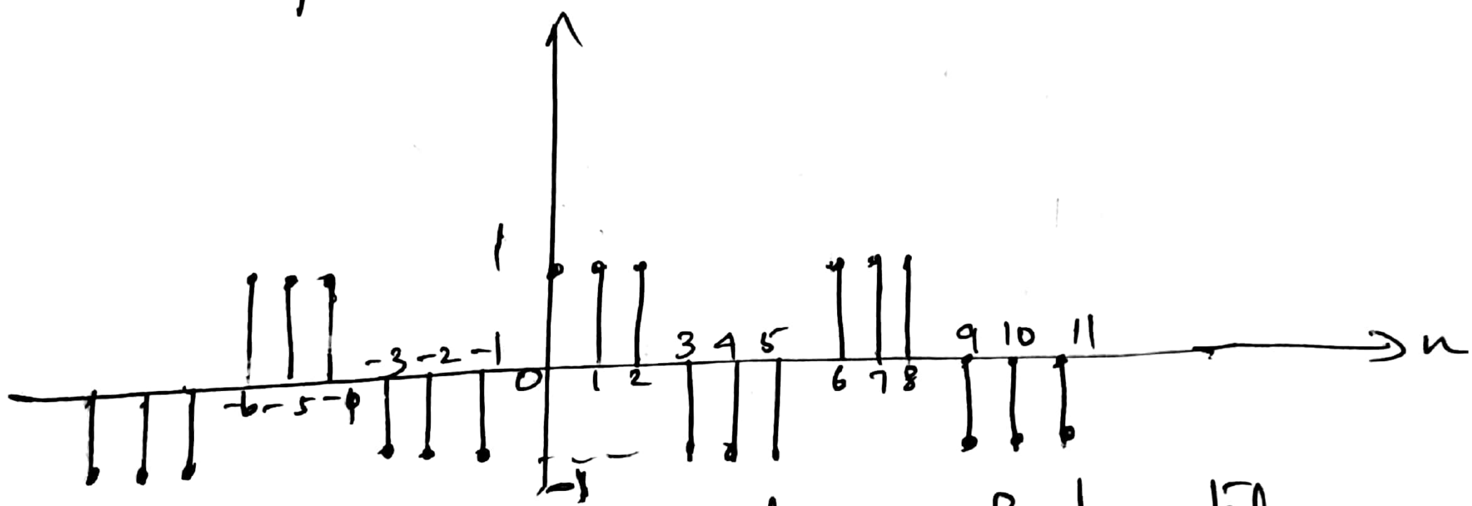
$$\boxed{x(n) = x(n+N)}$$

where, 'N' is a Positive Integer. The smallest value of 'N' which satisfies the above equation is called fundamental Period of the signal $x(n)$.

The fundamental angular frequency of $x(n)$ is

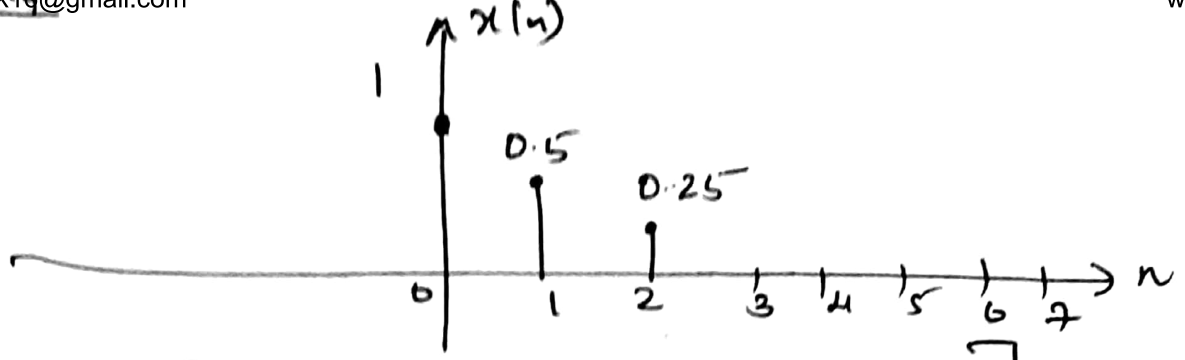
$$\Omega = \frac{2\pi}{N} \quad \text{(radians)}.$$

If $x(n) \neq x(n+N)$, then it is called non-Periodic or aperiodic signal



[A discrete-time Periodic Signal with fundamental Period N = 6].

Fig



[Discrete time non-Periodic Signal]

Q: 11 Determine whether the continuous time signal $x(t) = [\cos(2\pi t)]^2$ is Periodic or not. If Periodic, find the fundamental Period, T.

Sol^n

$$x(t) = [\cos(2\pi t)]^2$$

note $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \cos^2(2\pi t)$$

$\frac{1}{2} \Rightarrow dc$ (Periodic)

$$= \frac{1}{2}(1 + \cos 2(2\pi t))$$

$$x(t) = \frac{1}{2}(1 + \cos 4\pi t) \quad \text{--- (1)}$$

Compare eq^n (1) with standard form

$$\cos \omega_0 t$$

$$\omega_0 = 4\pi$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5 \text{ sec}$$

T = 0.5 is the fundamental time Period.

Verification :- (optional) Check $x(t) = x(t+T)$

$x(t) = x(t+0.5)$

$\underline{RHS} = x(t+0.5)$

We have, $x(t) = [\cos(2\pi t)]^2$

$x(t+0.5) = [\cos(2\pi(t+0.5))]^2$

$= \cos^2(2\pi(t+0.5))$

$= \dfrac{1+\cos 2(2\pi(t+0.5))}{2}$

$= \dfrac{1}{2}\left[1+\cos 4\pi(t+0.5)\right]$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \dfrac{1}{2}\left[1+\cos 4\pi t \cos 2\pi - \sin 4\pi t \sin 2\pi\right]$

$\cos 2\pi \rightarrow 1$, $\sin 2\pi \rightarrow 0$.

$= \dfrac{1}{2}\left[1+\cos 4\pi t\right]$

$= \dfrac{1}{2}\left[1+\cos 4\pi t\right] = \dfrac{1+\cos 2(2\pi t)}{2}$

$= \cos^2(2\pi t)$

$= (\cos(2\pi t))^2 = x(t)$

$\therefore x(t+T) = x(t)$

Hence, it is Periodic with Period $T = 0.5$ sec.

Q: 12 . Check for the Periodicity of $x(t) = 2\cos(3t + \pi/4)$

1.32

Sol^n: Comparing with $A\cos(\omega_0 t + \phi)$, we have the fundamental angular frequency

$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = 3 = \frac{2\pi}{T}$

$T = \frac{2\pi}{3}$ sec. Signal is Periodic with Period $2\pi/3$ sec.

Q: 13 Check for Periodicity of Signal $x(t) = e^{j\pi t}$

1.33

Sol^n Comparing with $e^{j\omega_0 t}$

$\omega_0 = \pi$

$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2\,sec$

$\therefore$ Signal is Periodic with Period 2 sec.

Q: 14 . Determine whether the Continuous time Signal $x(t) = \left[\sin(t - \pi/6)\right]^2$ is Periodic. If Periodic, find its fundamental Period.

Ans given, $x(t) = \sin(t - \pi/6)^2$

$= \sin^2(t - \pi/6)$

Note: $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\frac{1 - \cos 2(t - \pi/6)}{2} = \frac{1}{2} - \frac{1}{2}\cos(2t - \pi/3)$

$\cos(A - B)$

$= \cos A \cos B + \sin A \sin B$

$= \frac{1}{2} - \frac{1}{2}\left[\cos 2t \cos \pi/3 + \sin 2t \sin \pi/3\right]$

$= \frac{1}{2} - \frac{1}{2}\cos 2t \underbrace{\cos \pi/3}_{1/2} - \frac{1}{2}\sin 2t \underbrace{\sin \pi/3}_{\sqrt{3}/2}$

$= \underbrace{\frac{1}{2}}_{\text{(dc signal)}} - \frac{1}{4}\underbrace{\cos 2t}_{(\omega_0 = 2)} - \frac{\sqrt{3}}{4}\underbrace{\sin 2t}_{(\omega_0 = 2)}$

$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \underline{\underline{\pi \text{ sec}}}$.

$\therefore$ signal is Periodic with Period $\underline{\underline{\pi \text{ sec}}}$.

$\boxed{Q:15}$ $x(t) = \sin 6\pi t + \cos 5\pi t$

check the Periodicity of the signal and find its fundamental time Period.

Sol^n let $f(t) = \sin 6\pi t$

$\omega_{01} = 6\pi \qquad T_1 = \frac{2\pi}{\omega_{01}} = \frac{2\pi}{6\pi} = 2/6 = 1/3$

Let $g(t) = \cos 5\pi t$

$\omega_{o_2} = 5\pi$

$T_2 = \frac{2\pi}{\omega_{o_2}} = \frac{2\pi}{5\pi} = 2/5$

The ratio $\frac{T_1}{T_2} = \frac{1/3}{2/5} = 1/3 \times 5/2 = 5/6$ //.

Ratio $\frac{T_1}{T_2}$ is rational.

Hence the given signal is Periodic

To find fundamental time Period

$T_0 = \text{Lcm}(T_1, T_2)$

$= \text{Lcm}(1/3, 2/5)$

$= \frac{\text{Lcm}(1, 2)}{\text{HCF}(3, 5)} = \frac{2}{1} = 2\,\text{sec}$

Q: 16

$y(t) = \cos(3.5t) + \sin(2t) + 2\cos\left(\frac{7t}{6}\right)$

Sol^n

$y_1(t) = \cos(3.5t)$

$\omega_{01} = 3.5$

$T_1 = \frac{2\pi}{3.5} = \frac{2\pi}{7/2} = \frac{4\pi}{7}$ sec

$y_2(t) = \sin 2t$

$T_2 = \frac{2\pi}{2} = \pi$ sec

$y_3(t) = 2\cos\left(\frac{7t}{6}\right)$

$\omega_{03} = 7/6$

$T_3 = \frac{2\pi}{7/6}$

$= \frac{12\pi}{7}$

Ratio $\frac{T_1}{T_2} = \frac{\frac{4\pi}{7}}{\pi} = \underline{\underline{4/7}}$ (Rational)

$$\frac{T_1}{T_3} = \frac{4\pi/7}{12\pi/7} = \frac{4\pi}{\cancel{7}} \times \frac{\cancel{7}}{12\pi}$$

$$= \underline{\underline{1/3}} \quad \text{(Rational)}$$

As both $\frac{T_1}{T_2}$ & $\frac{T_1}{T_3}$ are rational, the given signal is <u>Periodic</u>

<u>To find fundamental Period</u> (T_0)

$$\frac{T_1}{T_2} = \frac{4}{7} \;;\; \frac{T_1}{T_3} = \frac{1}{3}$$

gcd of numerator = gcd(4,1) = $\underline{\underline{1}}$

gcd of denominator = gcd(7,3) = $\underline{\underline{1}}$

. Lcm of denominator = Lcm(7,3) = $\underline{\underline{21}}$.

Period of Sum of signals y(t)

= 21 (Period of $y_1(t)$).

$= 21(T_1)$

$= 21\left(\frac{4\pi}{7}\right)$

$$\boxed{T_0 = \underline{12\pi} \text{ sec}}$$

Q: (7) For the signal $y(t) = y_1(t) + y_2(t) + y_3(t)$, where $y_1(t)$, $y_2(t)$ and $y_3(t)$ have periods of 1.08, 3.6 and 2.025 sec. respectively.

Sol:- $T_1 = 1.08$; $T_2 = 3.6$, $T_3 = 2.025$

$$\frac{T_1}{T_2} = \frac{1.08}{3.6} = \frac{3}{10} \text{ (rational)}$$

$$\frac{T_1}{T_3} = \frac{1.08}{2.025} = \frac{8}{15} \text{ (rational)}$$

As both the ratios are rational, the given signal is periodic

To find the fundamental period ;

$$\frac{T_1}{T_2} = \frac{3}{10} \qquad \frac{T_1}{T_3} = \frac{8}{15}$$

note

$10 = 5 \times 2$
$15 = 5 \times 3$
$gcd = 5$

$$gcd(3, 8) = 1$$

$$gcd(10, 15) = 5$$

$$\therefore \frac{T_1}{T_2} = \frac{3(1)}{5(2)} \qquad \frac{T_1}{T_3} = \frac{8(1)}{5(3)}$$

Lcm of $(5, 3, 2) = 30$ //

$$\therefore T_0 = 30 \times T_1 = 30 \times 1.08$$

$$\boxed{T_0 = 32.4 \text{ sec}}$$

Q: 18 Determine whether the signal $x(t) = x_1(t) + 1.34\, x_2(t) + x_3(t)$ is periodic, where $x_1(t)$, $x_2(t)$ & $x_3(t)$ have periods of $8/3$, 1.26 & $\sqrt{2}$ sec respectively.

Sol^n $T_1 = \frac{8}{3}$ $T_2 = 1.26$ $T_3 = \sqrt{2}$

$$\frac{T_1}{T_2} = \frac{8/3}{1.26} = \frac{400}{189} \quad \text{(rational)}$$

$$\frac{T_1}{T_3} = \frac{8/3}{\sqrt{2}} = \frac{8}{3\sqrt{2}} \quad \text{(not rational)}$$

Since, $\frac{T_1}{T_3}$ cannot be brought to the form of ratio of two integers, therefore $x(t)$ is not periodic

Q: 19 Determine whether the signal $x(n) = (-1)^n$ is periodic. If periodic, find the fundamental period.

Sol^n $x(n) = (-1)^n$

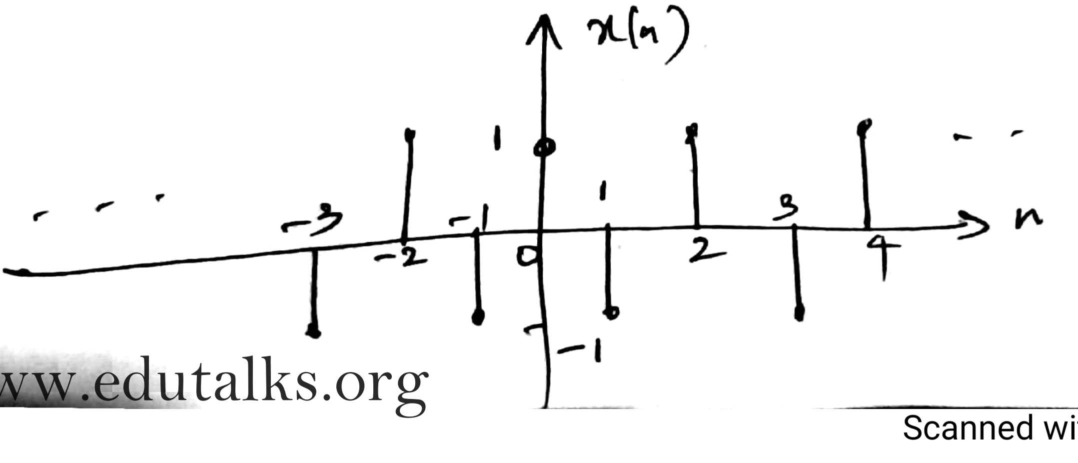
$x(0) = (-1)^0 = 1$

$x(1) = (-1)^1 = -1$

$x(2) = (-1)^2 = 1$

$x(-1) = (-1)^{-1} = \left(\frac{1}{-1}\right)^1 = (-1)^1 = \underline{-1}$

$x(-2) = (-1)^{-2} = \underline{(-1)^2} = 1$



By observation, the signal is ~~Sign~~ Periodic with Period $T_N = 2$ sec

1.43

$\boxed{Q:20}$. $x(n) = \cos\left(\frac{8\pi n}{7} + 2\right)$

Sol Compare with signal $\cos(\Omega n + \phi)$

$$\Omega = \frac{8\pi}{7}$$

$$N = \frac{2\pi}{\Omega} \cdot m$$

$$N = \frac{2\pi}{8\pi/7} \cdot m$$

$$= \frac{7}{4} m$$

for $m = 4$; $\boxed{N = 7}$.

note :- We need to give a Positive Integer to 'm' so as to get 'N' value.

$\therefore$ Fundamental Period $= 7$

$\boxed{Q:21}$ Determine whether the following signal is Periodic or not

$$x(n) = \underbrace{\left(\cos \frac{1}{3}\pi n\right)}_{x_1(n)} \underbrace{(\sin 2n)}_{x_2(n)}$$

$x_1(n) = \cos \frac{1}{3}\pi n$

$\Omega_1 = \pi/3$

$N_1 = \frac{2\pi}{\Omega_1} \cdot m_1$

$= \frac{2\pi}{\pi/3} \cdot m_1$

$= 6\, m_1$

for $m_1 = 1$; $\boxed{N_1 = 6}$

$x_2(n) = \sin 2n$

$\Omega_2 = 2$

$N_2 = \frac{2\pi}{\Omega_2} \cdot m_2$

$= \frac{2\pi}{2} \cdot m_2$

$= \pi m_2$

for any 'm_2'

N_2 cannot be an Integer.

$\therefore$ $x_2(n)$ is not Periodic

Note :-

Multiplying any Periodic signal with non-Periodic signal results in non-Periodic signal.

1.4]

$\boxed{Q:22}$. Determine if $x(n) = \cos\left(\frac{1}{5}\pi n\right) \sin\left(\frac{1}{3}\pi n\right)$ is Periodic or not.

Sol:

$x(n) = \cos\left(\frac{1}{5}\pi n\right) \sin\left(\frac{1}{3}\pi n\right)$

Multiplying & Dividing by 2

$\frac{1}{2}\left[2 \cos\left(\underbrace{\frac{1}{5}\pi n}_{B}\right) \sin\left(\underbrace{\frac{1}{3}\pi n}_{A}\right)\right]$

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$= \frac{1}{2}\left[\sin\left(\frac{1}{5}\pi n + \frac{1}{3}\pi n\right) + \sin\left(\frac{1}{3}\pi n - \frac{1}{5}\pi n\right)\right.$

$$= \frac{1}{2}\left[\sin\frac{8\pi n}{15} + \sin\frac{2\pi n}{15}\right]$$

$$= \underbrace{\frac{1}{2}\sin\frac{8\pi n}{15}}_{x_1(n)} + \underbrace{\frac{1}{2}\sin\frac{2\pi n}{15}}_{x_2(n)}$$

<u>for $x_1(n)$</u>

$\Omega_1 = \frac{8\pi}{15}$

$N_1 = \frac{2\pi}{\Omega_1} m_1$

$= \frac{2\pi}{\frac{8\pi}{15}} m_1$

$N_1 = \frac{15}{4} m_1$

for $m_1 = 4$

$\boxed{N_1 = 15}$

$\Omega_2 = 2\pi/15$

$N_2 = \frac{2\pi}{\Omega_2} \cdot m_2$

$= \frac{2\pi}{\frac{2\pi}{15}} \cdot m_2$

$= 15 m_2$

for $m_2 = 1$

$\boxed{N_2 = 15}$

Ratio $\frac{N_1}{N_2}$ is $\frac{15}{15} = 1$

is a rational no

Hence <u>it is Periodic</u>

$N_0 = LCM(N_1, N_2) = Lcm(15, 15) = 15$ //

$\therefore$ Fundamental time Period

$= \underline{15 \text{ sec}}$

$15 \,|\, 15, 15$
$\quad 1, 1$
$= 15$ //

Q: 22. Find the Periodicity of the signal.

$$x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$$

Sol^n $\Omega_1 = \frac{2\pi}{5}$; $\Omega_2 = \frac{2\pi}{7}$

$$N_1 = \frac{2\pi}{\Omega_1} m_1$$

$$= \frac{2\pi}{2\pi/5} \cdot m_1$$

$$N_1 = 5 m_1$$

for $m_1 = 1$; $\underline{N_1 = 5}$

$$N_2 = \frac{2\pi}{\frac{2\pi}{7}} m_2$$

$$= 7 m_2$$

for $m_2 = 1$

$\underline{N_2 = 7}$

$\frac{N_1}{N_2} = \frac{5}{7}$ (rational)

Hence it is Periodic

$N_0 = \mathrm{Lcm}(5, 7) = \underline{35}$

Fundamental time Period = $\underline{35 \text{ sec}}$.

1.42 Q: 23. Check whether the following signals are Periodic or not. If Periodic find the fundamental Period.

(i) $x_1(n) = \cos 2\pi n$ (ii) $x_2(n) = \cos 2n$.

Sol^n (i) $x_1(n) = \cos 2\pi n$

$$\Omega_1 = 2\pi$$

$$N_1 = \frac{2\pi}{\Omega_1} m_1 = \frac{2\pi}{2\pi} m_1 = m_1$$

for $m_1 = 1$; $\underline{N_1 = 1 \text{ sec}}$.

Signal is Periodic with $\underline{\text{Period } 1 \text{sec}}$.

(ii) $x_2(n) = \cos 2n$.

kashok16@gmail.com

$\Omega_2 = 2$

$\rightarrow N_2 = \frac{2\pi}{\Omega_2} \cdot m_2 = \frac{2\pi}{2} \cdot m_2$

$N_2 = \pi m_2$

for any value of m_2 N_2 cannot be Integer.

$\therefore x_2(n)$ Cannot be Periodic

(iii) Deterministic and Random Signals.

A deterministic signal behaves in a fixed known way with respect to time. It can be modelled as a function of time 't' (ie, Continuous time signal) or a function of sample number 'n' (ie, discrete time signal)

A Random signal takes on one of several possible values at each time for which a signal value is defined. It is a signal about which there is uncertainty with respect to its value at any time.

eg: noise, ECG signal.

(iv) Energy & Power Signal

→ The signal is referred to as energy signal, If the total energy 'E' of the signal satisfies the condition

$$0 < E < \infty \quad (\text{i.e, } 'E' \text{ must be finite})$$

$$P = 0 \quad (\text{Power is zero}).$$

→ The signal is said to be **Power signal**, if the average Power 'P' of the signal satisfies the condition,

$$0 < P < \infty \quad (\text{i.e, } 'P' \text{ must be finite})$$

$$E = \infty \quad (\text{i.e, Energy is Infinity}).$$

Formulas to Calculate Energy & Power:-

For Continuous time Signal:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt$$

For discrete time signal:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Q 24 Check whether the following signal $x(t)$ is energy or Power signal & find the corresponding value.

$$x(t) = \begin{cases} t \ ; & 0 \le t \le 1 \\ 2-t \ ; & 1 \le t \le 2 \\ 0 \ ; & \text{otherwise.} \end{cases}$$

Sol

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_0^2 |x(t)|^2 dt = \int_0^1 t^2 dt + \int_1^2 (2-t)^2 dt$$

$$\int_1^2 4 + t^2 - 4t$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[4t + \frac{t^3}{3} - \frac{4t^2}{2}\right]_1^2$$

$$= \frac{1}{3} - 0 + \cancel{8} + \frac{8}{3} - \cancel{8} - \left[4 + \frac{1}{3} - 2\right] = \frac{8}{3} - 2$$

$$= \cancel{\frac{2}{3}\text{ Joule} < \infty} \qquad = \frac{2}{3}\text{ Joule}$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[5/6\right] = \lim_{T \to \infty} \frac{5}{6T} =$$

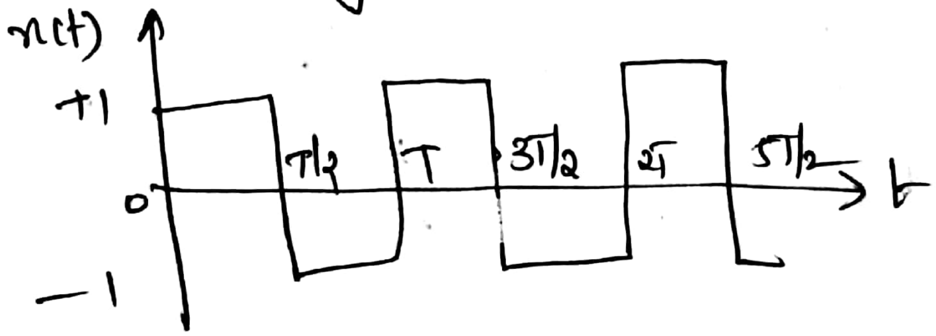
$$= \frac{5}{\infty} = 0$$

'E' is finite & 'P' is zero

$\therefore$ Signal is Energy signal

1.48

$\boxed{Q: 25}$. What is the average Power of the Square Wave shown in figure?



Sol: Here fundamental Period $= T$

$$x(t) = 1; \quad 0 < t < T/2$$
$$= -1; \quad T/2 < t < T$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \int_{0}^{T} x^2(t)\, dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\int_{0}^{T/2} 1^2\, dt + \int_{T/2}^{T} (-1)^2\, dt \right]$$

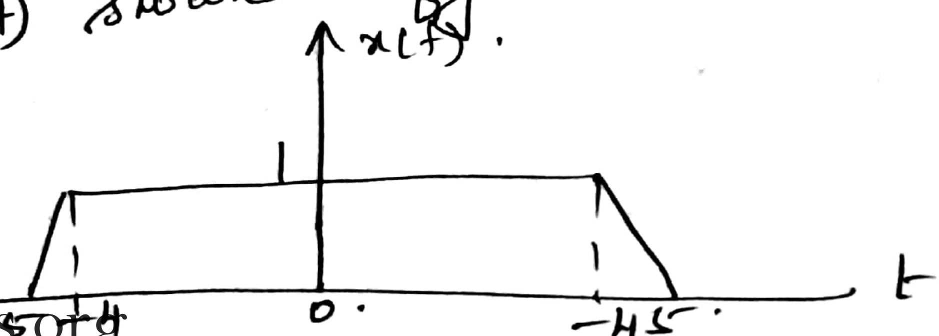
$$\underset{T\to\infty}{Lt} \frac{1}{T} \left[(t)_0^{T/2} + (t)_{T/2}^{T} \right]$$

$$\underset{T\to\infty}{Lt} \frac{1}{T} \left[T/2 - 0 + T - T/2 \right]$$

$$\underset{T\to\infty}{Lt} \frac{1}{T} \left[T \right] = 1 \text{ watts}$$

1:52

$\boxed{Q:26}$. Find the total energy for the trapezoidal Pulse $x(t)$ shown in figure:



$$x(t) = 5-t \; ; \; 4 \le t \le 5$$
$$= 1 \; ; \; -4 \le t \le 4$$
$$= t+5 \; ; \; -5 \le t \le -4$$
$$= 0 \; ; \; \text{Otherwise.}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt =$$

$$= \int_{-5}^{-4} (t+5)^2 dt + \int_{-4}^{4} 1^2 dt + \int_{4}^{5} (5-t)^2 dt$$

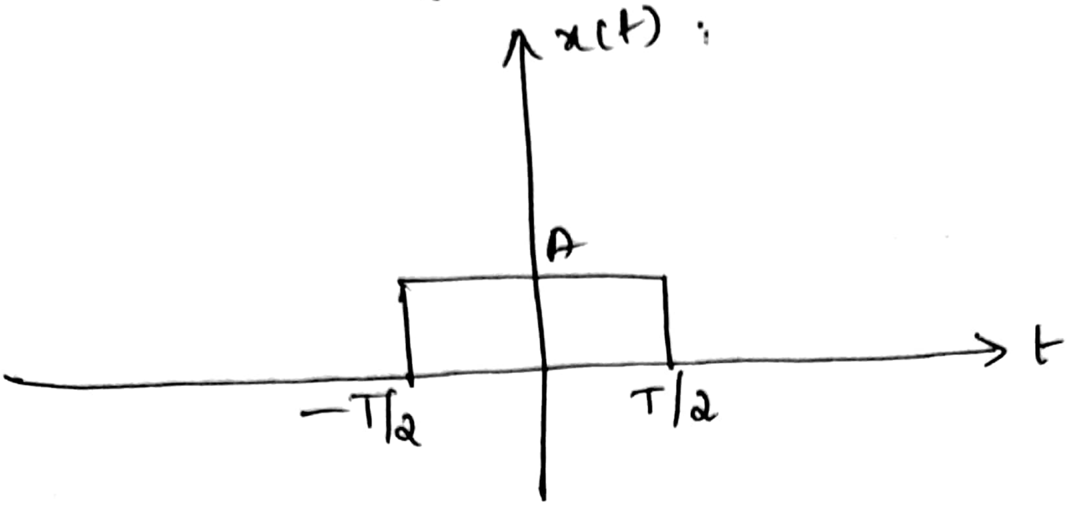
$$= \int_{-5}^{-4} (t^2 + 10t + 25) + (t)_{-4}^{4} + \int_{4}^{5} (25 - 10t + t^2) dt$$

$$= \left(\frac{t^3}{3} + \frac{10t^2}{2} + 25t\right)_{-5}^{-4} + (4+4)$$
$$+ \left[25t - 10\frac{t^2}{2} + \frac{t^3}{3}\right]_4^5$$

$$\Rightarrow \frac{-64}{3} + 80 - 100 \cancel{+8} + \frac{125}{3} + \frac{250}{2}$$
$$-125 + 8 + 125 - \frac{250}{2} + \frac{125}{3} - 100$$
$$+ 80 - \frac{64}{3} = \frac{26}{3} \text{ Joule}$$

1.4.1

Q: 28 What is the total energy of the rectangular pulse shown in figure?



Soln

$x(t) = A \; ; \; T/2 \le t \le T/2$

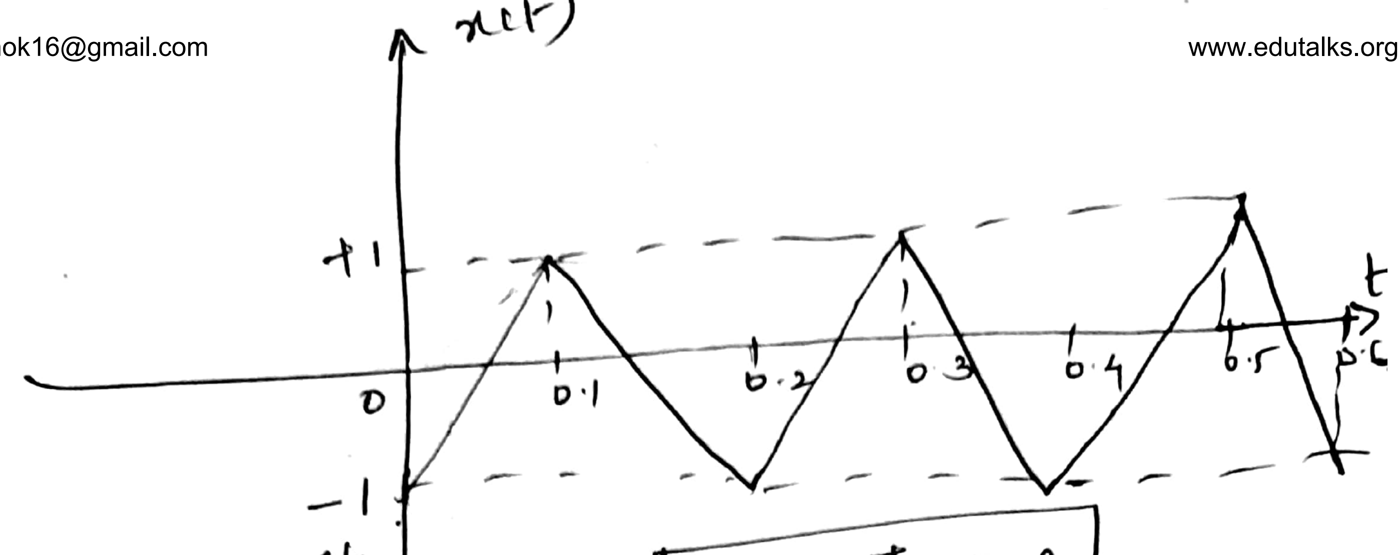
$\quad\;\; = 0 \; ; \;$ otherwise

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-T/2}^{T/2} A^2 dt = A^2 [t]_{-T/2}^{T/2}$$

$$= A^2 \left[T/2 + T/2 \right] = \underline{\underline{A^2 T}} \text{ Joule.}$$

Q: 29 1.5.1

What is the average power of the triangular wave shown in figure:-



$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$$ $\boxed{\text{Here, } T = 0.2}$

$x(t) = 20t - 1 \;; \quad 0 \le t \le 0.1$

$\quad = -20t + 3 \;; \quad 0.1 \le t \le 0.2$

$$P = \lim_{T\to\infty}\frac{1}{T}\left[\int_0^{0.1}(20t-1)^2\, dt + \int_{0.1}^{0.2}(-20t+3)^2\, dt\right]$$

$$= \frac{1}{0.2}\left[\int_0^{0.1}(400t^2 + 1 - 40t)\, dt + \int_{0.1}^{0.2}(400t^2 + 9 - 120t)\, dt\right].$$

note

$y = mx + c$

$1 = m\,0.1 + c$

$-1 = m \cdot 0 + c$

$\underline{c = -1}$

$1 = m \cdot 0.1 - 1$

$m(0.1) = 2$

$m = \frac{2}{0.1} = \underline{20}$

$\therefore y = \underline{20t - 1}$

$1 = m\,0.1 + c$

$-1 = 0.2m + c$

$2 = -0.1m$

$\underline{m = -20}$

$1 = -2 + c$

$\underline{c = 3}$

$y = -20t + 3$

$$= \frac{1}{0.2}\left[\frac{400t^3}{3} + t - \frac{40t^2}{2}\right]_0^{0.1} + \frac{1}{0.2}\left[\frac{400t^3}{3} + 9t - \frac{120t^2}{2}\right]_{0.1}^{0.2}$$

$$= \frac{1}{0.2}\left[\frac{400(0.1)^3}{3} + 0.1 - 20(0.1)^2\right] +$$

$$\frac{1}{0.2}\left[\frac{400(0.2)^3}{3} + 9(0.2) - 60(0.2)^2 - \frac{400(0.1)^3}{3} - 9(0.1) - \cancel{400}\ 60(0.1)^2\right]$$

$$= \frac{1}{0.2}\left[\frac{2}{15} + 0.1 - \frac{1}{5} + \frac{16}{15} + \frac{9}{5} - \frac{12}{5} - \frac{2}{15} - \frac{9}{10} + \frac{3}{5}\right]$$

$$= \frac{1}{3} \text{ Watts}$$

Q(29) Consider the signal given by $x(t) = A\cos(\omega t + \phi)$. Determine the average Power of it.

15M

Sol^n Average Power, $P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A^2 \cos^2(\omega t + \phi)\, dt$$

$$= \frac{A^2}{2T}\int_{-T/2}^{T/2} \left[1 + \cos 2(\omega t + \phi)\right] dt$$

$$= \frac{A^2}{2T}\int_{-T/2}^{T/2} 1\, dt + \underbrace{\frac{A^2}{2T}\int_{-T/2}^{T/2} \cos 2(\omega t + \phi)}_{\text{Integral over a Closed Interval is zero for a Cosine wave}} \rightarrow 0$$

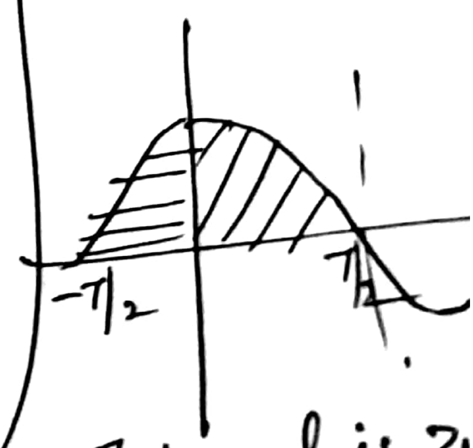
$$\Rightarrow \frac{A^2}{2T}\left[t\right]_{-T/2}^{T/2}$$

$$= \frac{A^2}{2T}\left[T/2 + T/2\right]$$

$$= \frac{A^2}{2T}\left[T\right]$$

$$\boxed{P = \frac{A^2}{2} \text{ Watts}}$$

note



Integral is ze

$= A_0$ watts

$\boxed{Q: 32}$. Check whether the following signal $x(n)$ is energy or Power signal and find the corresponding value.

1.57

$$x(n) = \begin{cases} n & ; \quad 0 \le n \le 5 \\ 10-n & ; \quad \cancel{6} \le n \le 10 \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Sol^n

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{10} |x(n)|^2$$

$$= \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2$$

$$= 0^2+1^2+2^2+3^2+4^2+5^2 + (10-6)^2 + (10-7)^2 + (10-8)^2 + (10-9)^2 + (10-10)^2$$

$$= \underline{\underline{85 \text{ Joule}}} \quad (\text{finite value})$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \underbrace{\sum_{n=0}^{10} |x(n)|^2}_{85}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (85) = \frac{85}{\infty} = \underline{\underline{0 \text{ watts}}}$$

As Energy is finite & Power is zero, the given signal is known as <u>energy signal</u>.

$\boxed{Q:33}$ check whether the following are energy or Power signals?. Also find the Corresponding values.

(i) $x(n) = \cos\pi n \;;\; -4 \le n \le 4$
$\quad\quad = 0 \;;\;$ otherwise.

<u>Soln</u> $E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-4}^{4} (\cos\pi n)^2$

$$= \sum_{n=-4}^{4} \left((-1)^n\right)^2$$

$$\sum_{n=-4}^{4}\left[(-1)^2\right]^n$$

$$= \sum_{n=-4}^{4}(1)^n = \underbrace{1+1+1+1 \cdots +1}_{9 \text{ terms}}$$

$$E = \underline{9} \text{ Joule}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \underbrace{\sum_{n=-4}^{4} (\cos \pi n)^2}_{9}$$

$$= \lim_{N\to\infty} \frac{9}{2N+1} = \frac{9}{\infty} = \underline{0 \text{ watts}}$$

As energy is finite & Power is zero, the given signal is known as <u>energy signal</u>.

$\boxed{Q:34}$ Check whether the following are energy or Power signal & Also, find the corresponding value.

$$x(n) = \cos \pi n \;; \; n \geq 0$$
$$= 0 \;; \; \text{otherwise}.$$

Sol

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} (\cos \pi n)^2$$

$$= \sum_{n=0}^{\infty} \left[(-1)^n\right]^2 = \sum_{n=0}^{\infty} \left((-1)^2\right)^n = \sum_{n=0}^{\infty} 1^n.$$

$$= 1+1+1+\cdots 1$$

$$= \underline{\underline{\infty}}.$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left((-1)^2\right)^n \Rightarrow \lim_{N\to\infty} \frac{1}{2N+1}(N+1)$$

$$= \lim_{N\to\infty} \frac{\cancel{N}(1+1/N)}{2\cancel{N}(1+\frac{1}{2N})}$$

$$P = \frac{1}{2}\left(\frac{1+0}{1+0}\right) = \underline{\underline{1/2}} \text{ Watts}.$$

As, Power is finite & energy is Infinite, the given signal is known as Power signal.

II Elementary signals / Functions.

Many Physical signals that occur in nature can be modelled using elementary signals. Some of the Important (A) elementary - Continuous time signals are:-

(i) Unit step function

(ii) Unit Impulse function

(iii) Unit Ramp function.

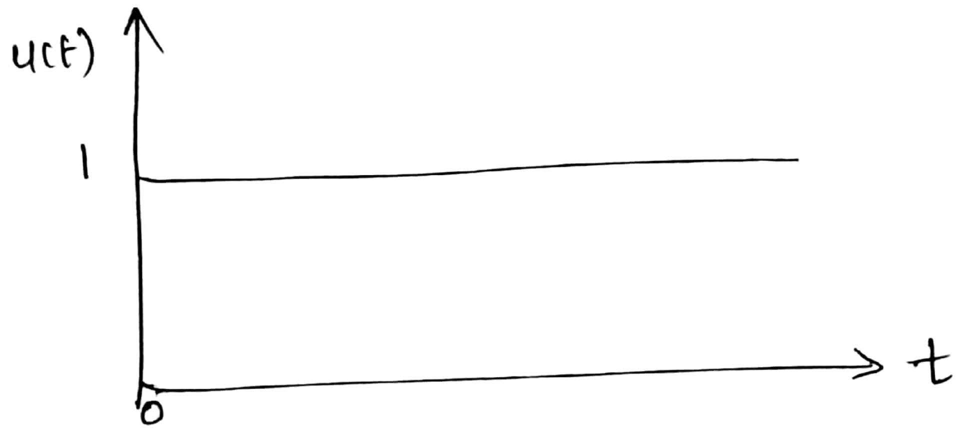
(iv) Sinusoidal Signals.

(v) Exponential Signals.

(i) Unit Step function $[u(t)]$

The Continuous-time unit step function is defined as,

$$u(t) = 1 \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$



(ii) unit Impulse function : $\delta(t)$

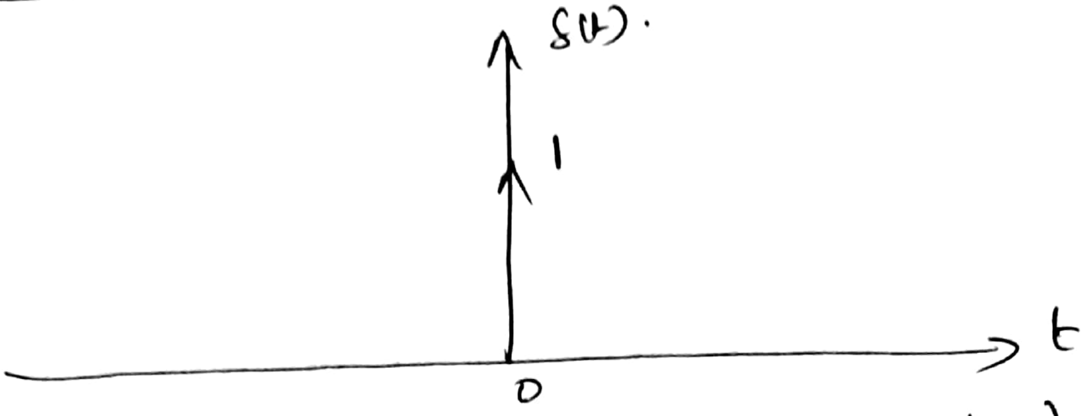
The Continuous-time unit Impulse function $\delta(t)$ is defined as

$$\delta(t) = 0 \quad t \neq 0$$

It only exist at $t = 0$

for unit Impulse function, $\delta(t) = 1$ at $t = 0$.

This function $\delta(t)$ is known as Dirac delta function.



(Unit Impulse function)

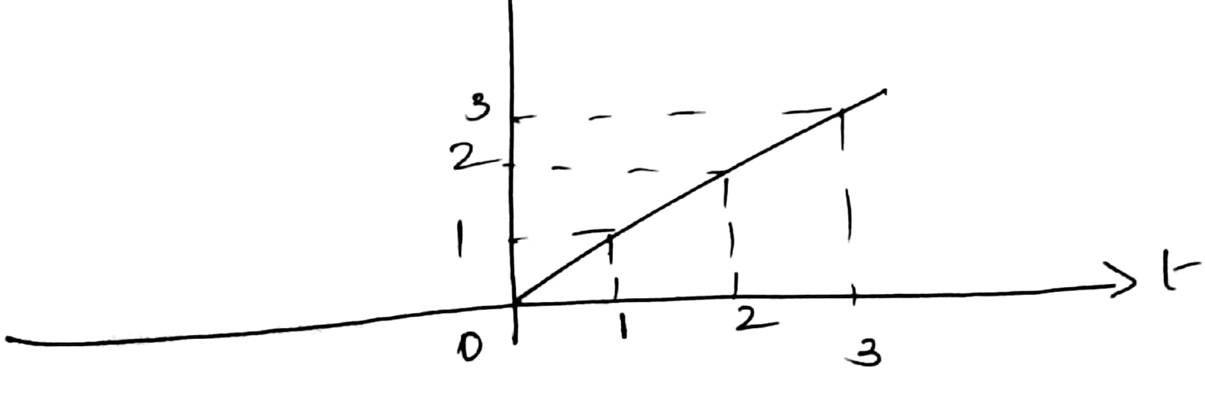
We have $$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

ie, area covered by an unit Impulse function is unity.

An Impulse function has.

(a) Zero width
(b) Infinite height
(c) Unit area / unit strength.

(iii) Unit Ramp function :-



A ramp function is defined as,

$$\boxed{\begin{aligned} r(t) &= t \; ; \; t \geq 0 \\ &= 0 \; ; \; \text{otherwise.} \end{aligned}}$$

It is Integral of the Unit step function $u(t)$.

$$\int u(t)\, dt = \int 1\, dt = \underline{\underline{t}}$$

(IV) <u>Sinusoidal Signal</u>

A sinusoidal signal is given by :-

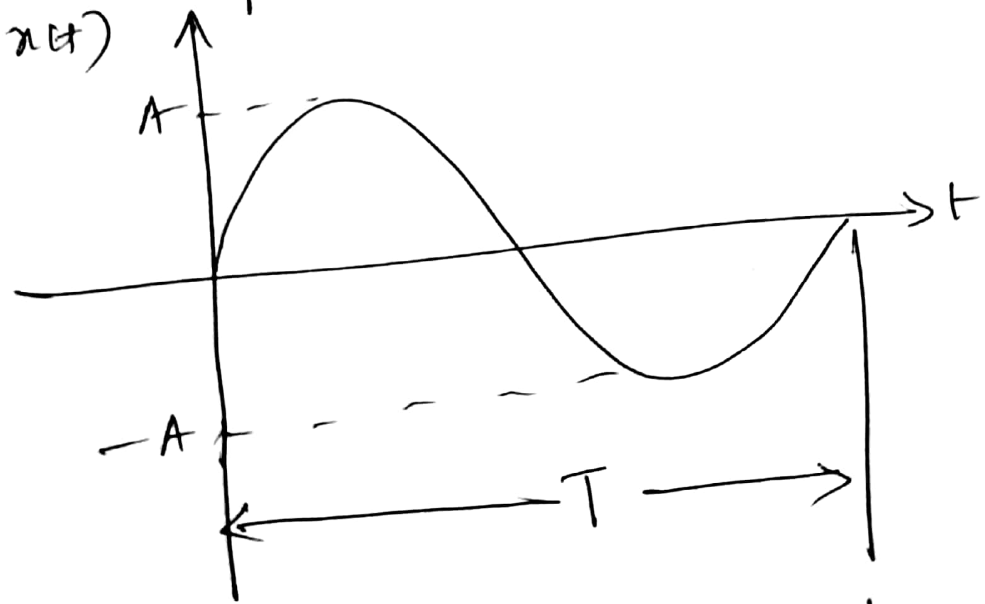
$$x(t) = A \sin(\omega_0 t + \phi)$$

where,

$\omega_0 = 2\pi f_0$ = Angular frequency (rad/sec)

f_0 = Linear frequency (Hz)

ϕ = phase shift (radians)



where, $T = \frac{2\pi}{\omega_0} = \frac{1}{f_0}$. $\quad T \rightarrow$ fundamental time Period.

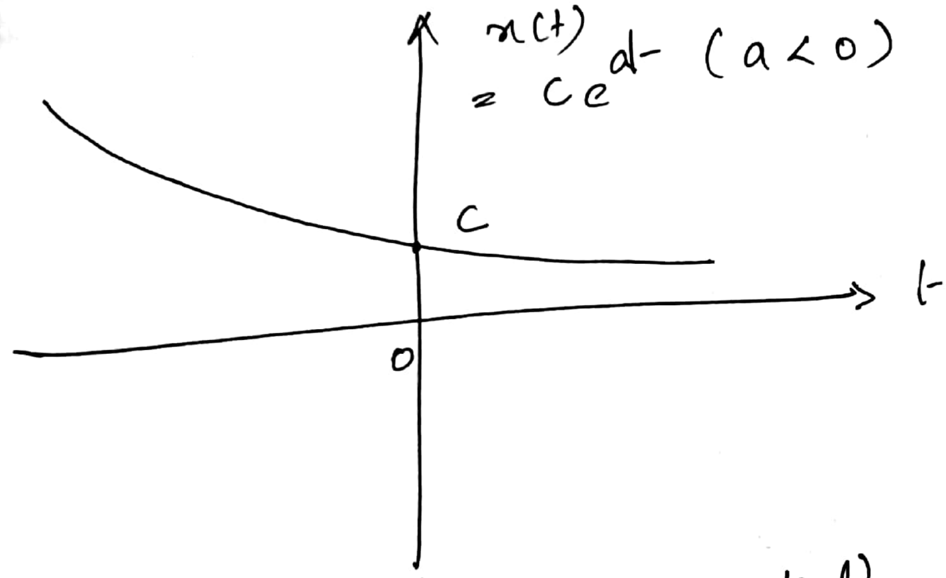
(v) Exponential Signals.

A real exponential Continuous-time signal is given as

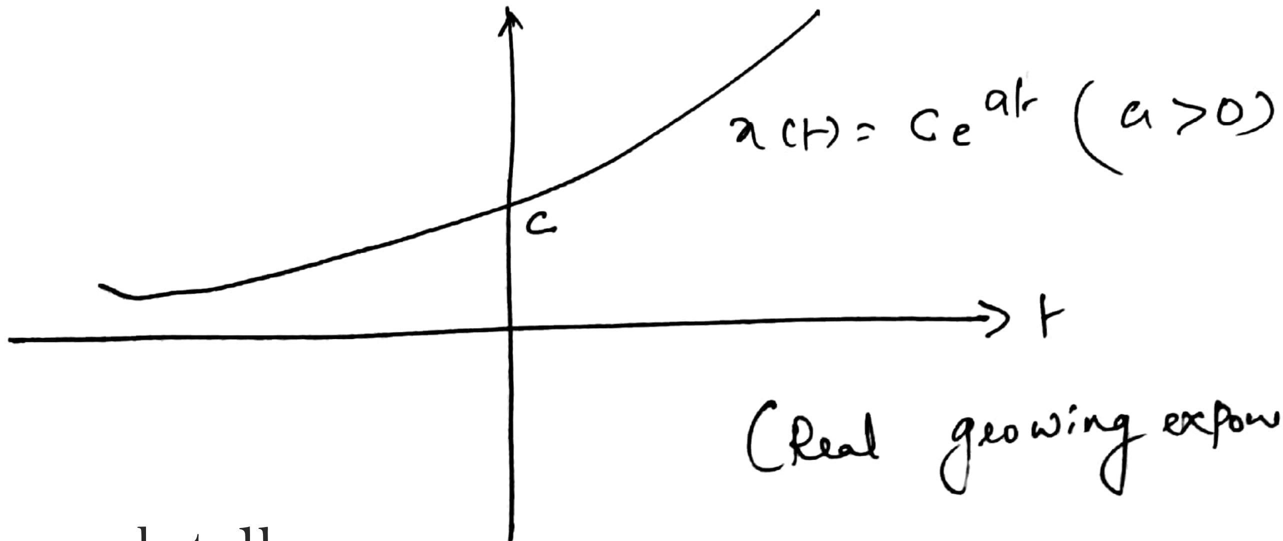
$$x(t) = Ce^{at}.$$

where both 'C' and 'a' real Constant. 'C' is known as the amplitude of the exponential signal at $t=0$.

* If $a<0$, (ie, 'a' is negative), the signal $x(t)$ is known as decaying exponential signal, and If $a>0$ (ie, 'a' is positive) then $x(t)$ is called growing exponential signal.



(Real decaying exponential).



(Real growing exponential).

Complex - exponential Signal.

If 'c' or 'a' or both are Complex numbers, then $x(t)$ is known as Continuous-time exponential Signal.

In eq^n $x(t) = Ce^{at}$

Consider, $C = 1$ and 'a' is Imaginary.

ie, $x(t) = e^{j\omega_0 t}$ → Rising Complex exponential signal

If $x(t) = e^{-j\omega_0 t}$ → decaying Complex exponential signal.

(B) Elementary discrete time Signals.

The basic important discrete time signals are:-

(i) Unit Step Sequence.

(ii) Unit Impulse Sequence.

(iii) Unit Ramp Sequence.

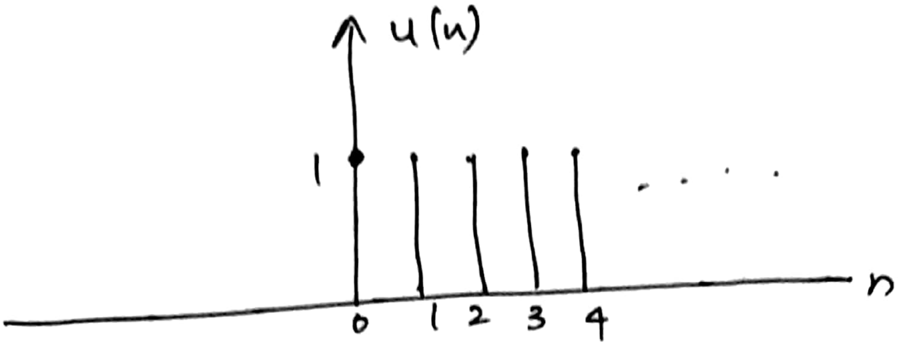
(iv) Sinusoidal Signal.

(v) Exponential Signal.

(i) Unit Step Sequence: $[u(n)]$

A discrete-time unit step Sequence is defined as:-

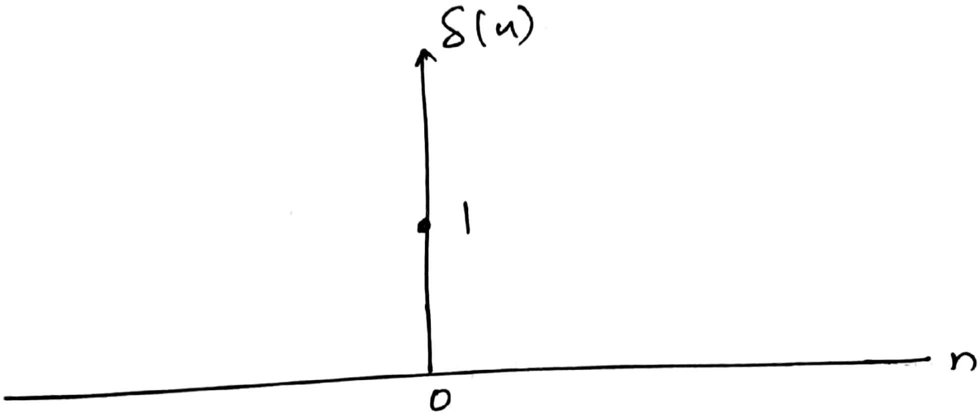
$$u(n) = 1 \;;\; n \geq 0$$
$$= 0 \;;\; n < 0$$



(ii) Unit - Impulse Sequence : ($\delta(n)$)

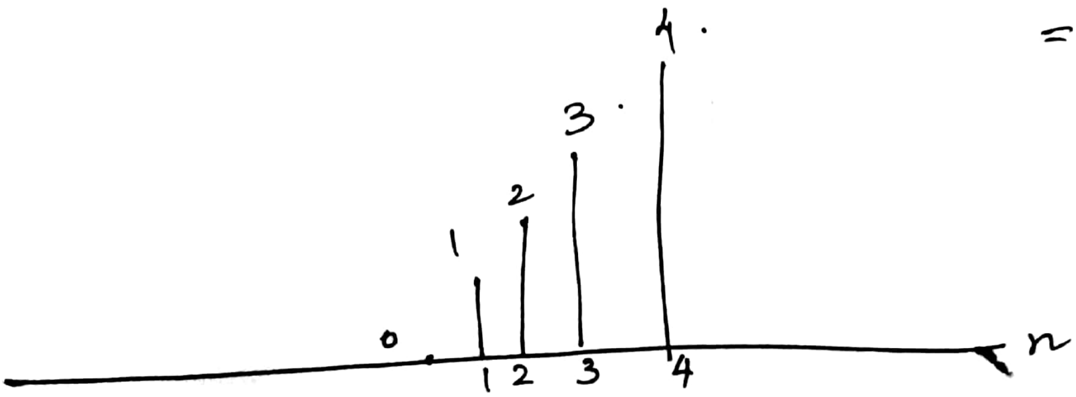
A discrete - time Impulse Sequence is defined as

$$\delta(n) = 1 ; \quad n = 0$$
$$= 0 ; \quad n \neq 0$$



(iii) Discrete - time Unit- ramp Sequence :

$$r(n) = n ; \quad n \geq 0$$
$$= 0 ; \quad n < 0$$



(iv) Discrete - time Sinusoidal Signal.

A discrete - time version of a sinusoidal signal is given by :

$$x(n) = A \cos (\Omega_0 n + \phi)$$

'A' — Max. Value of $x(n)$.

Ω_0 – Angular frequency.

ϕ – Phase Angle.

'Ω_0' & 'ϕ' are measured in radians.

'n' – dimensionless quantity.

where $\Omega_0 = \frac{2\pi}{N} . m$.

where, 'm' and 'N' are integers.

eg:- Let $x(n) = \sin (\pi/4 \, n)$

Comparing with $A \sin (\Omega_0 n + \phi)$

$A = 1$

$\Omega_0 = \pi/4 \Rightarrow N = \frac{2\pi}{\pi/4} m$.

$\phi = 0$

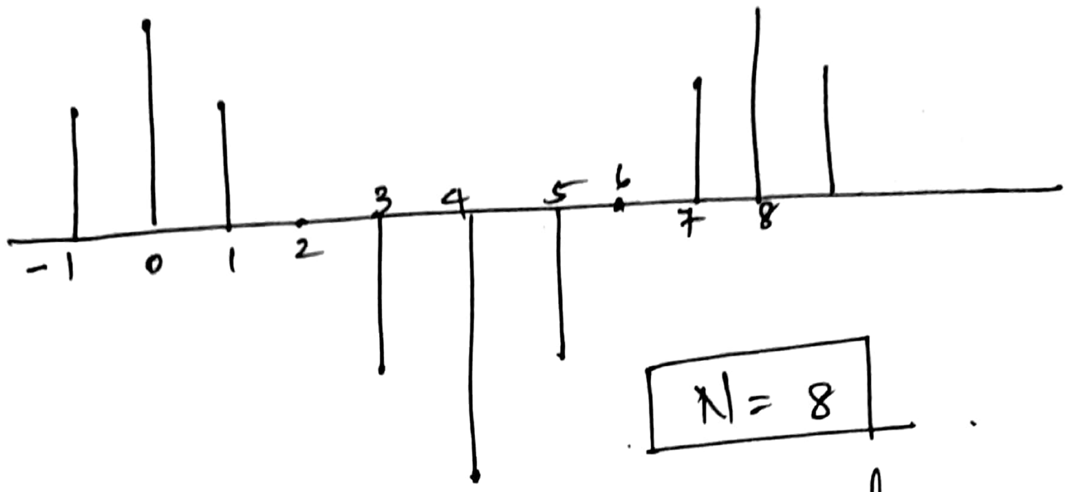
$N = 8m$

$m = 1$

$N = 8$

$\therefore$ Fundamental Period = 8.

Plot for $x(n) = \cos\left(\frac{\pi n}{4}\right)$



(v) Discrete - time Exponential signal.

$$\boxed{x(n) = C\alpha^{n}}$$

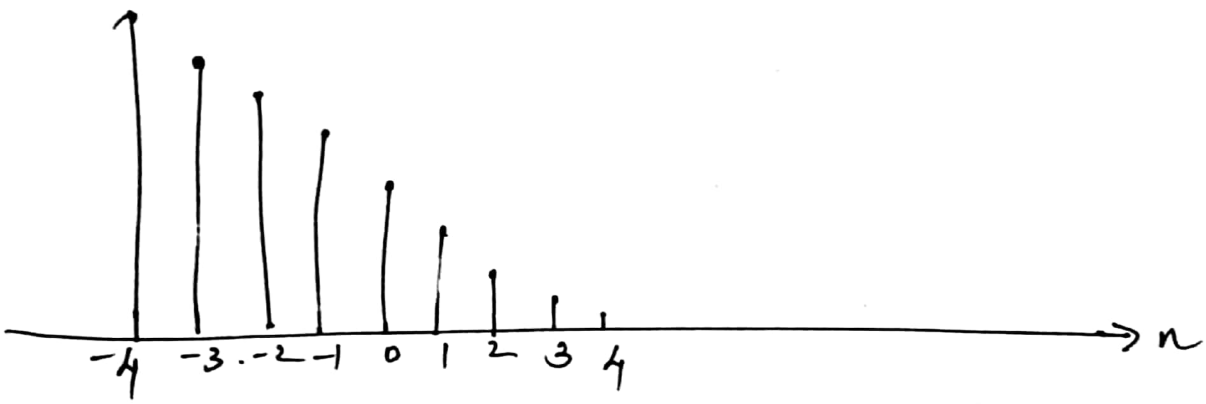
where, $\alpha = e^{\beta}$

C, α and β are real constants. 'C' is the amplitude of the sequence at $n = 0$.

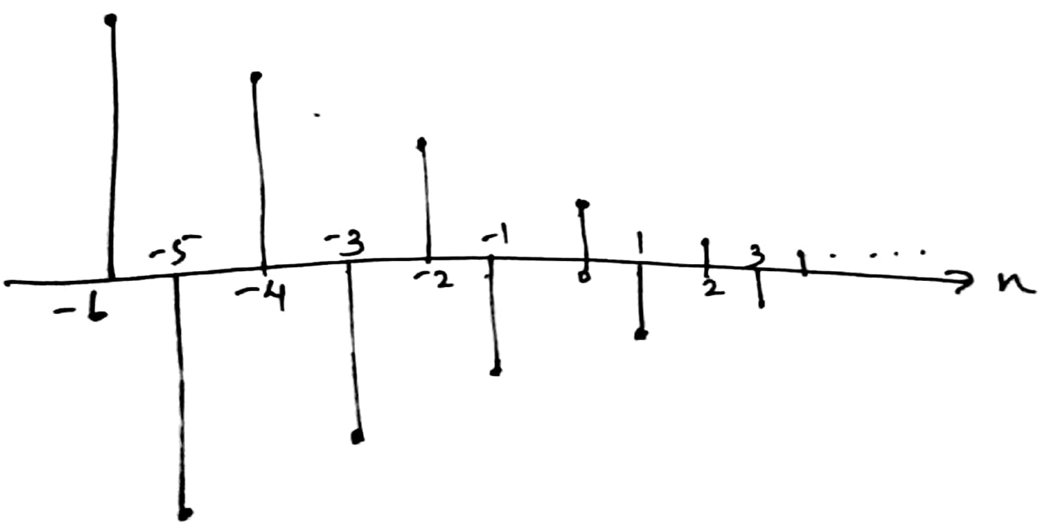
* If $|\alpha| < 1$, the signal decays exponentially.
* If $\alpha < 0$ (α is negative), then sign of $x(n)$ alternates $[-1 < \alpha < 0]$
* When $\alpha > 1$, the signal grows exponentially.
* When $\alpha < -1$, sign of $x(n)$ alternates in growing manner.

Case: 1 $[\alpha > 1]$

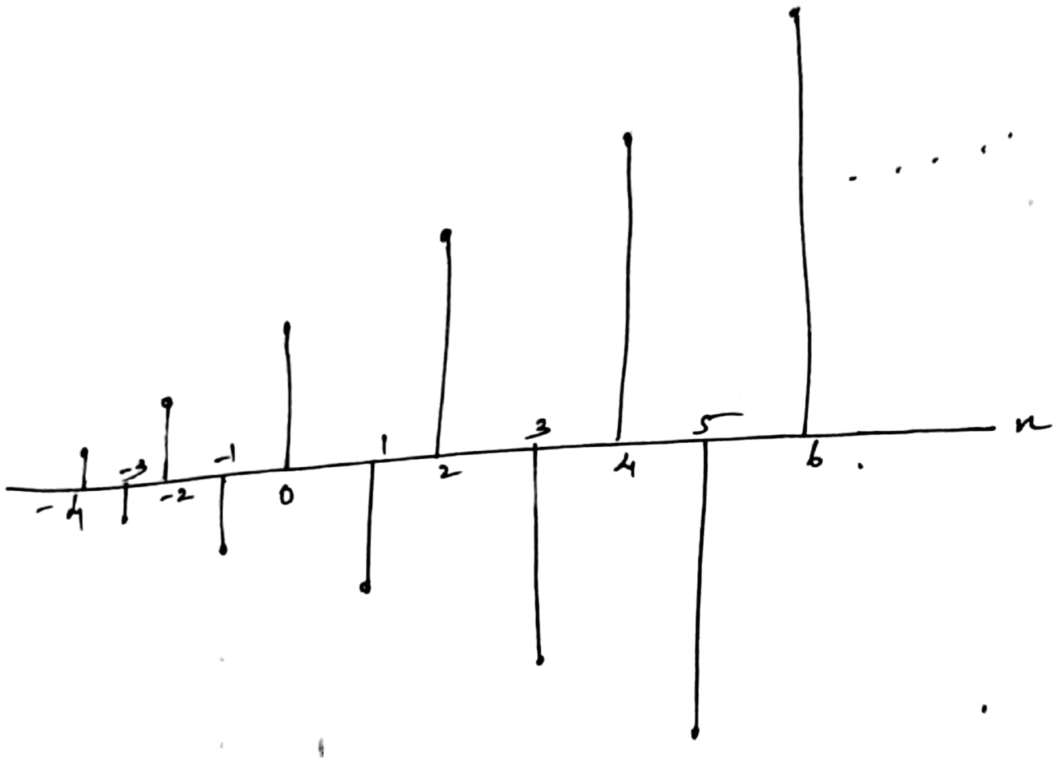
Case: 2 $[0 < \alpha < 1]$



Case: 3 $[-1 < \alpha < 0]$



Case 4: $\boxed{\alpha < -1}$.



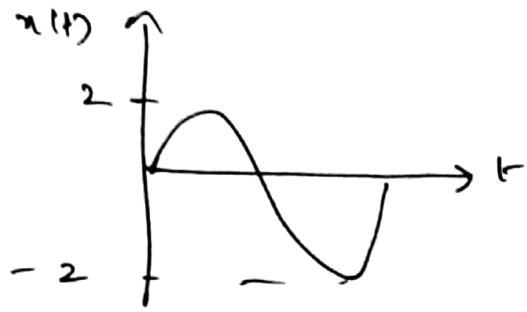
III Basic operations on Signals:

1) Amplitude Scaling:-

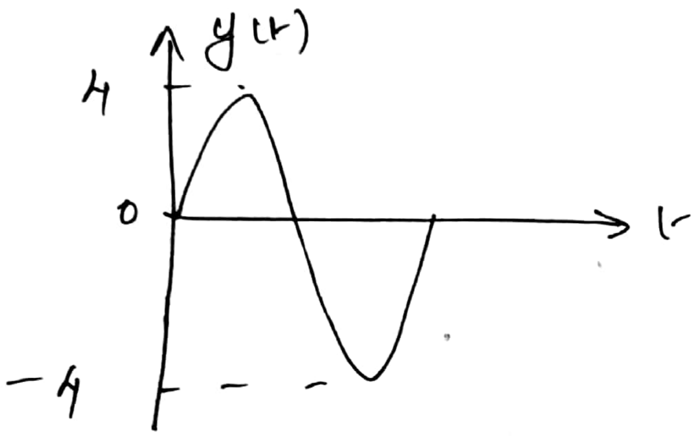
* Let $x(t)$ be a continuous time signal. Then the signal $\boxed{y(t) = Cx(t)}$ is known as amplitude scaled version of $x(t)$ where 'c' is known as scaling factor. The signal $y(t)$ is obtained by multiplying the amplitude of $x(t)$ by scalar 'c' at all time 't'.

* Similarly let $x(n)$ be a discrete time signal. Then the signal $\boxed{y(n) = Cx(n)}$ is known as scaled version of $x(n)$.

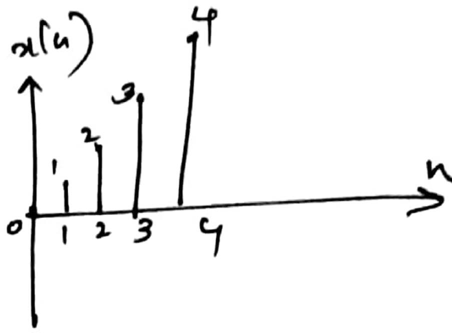
eg:- Let $x(t)$ is given by.



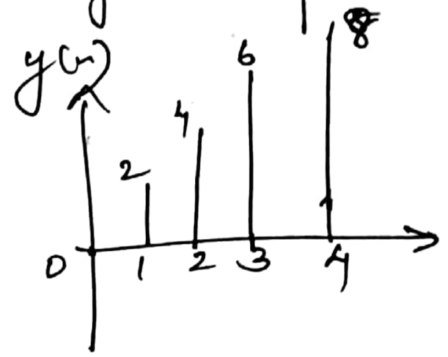
$y(t) = 2x(t)$ is shown as:-



Let $x(n)$ is given by



$y(n) = 2x(n)$ is given by:-



(ii) <u>Time Scaling</u>:-

→ Let $x(t)$ be a continuous-time signal. The signal $y(t)$ is obtained by scaling the independent variable 't' by a factor 'a' is given by:-

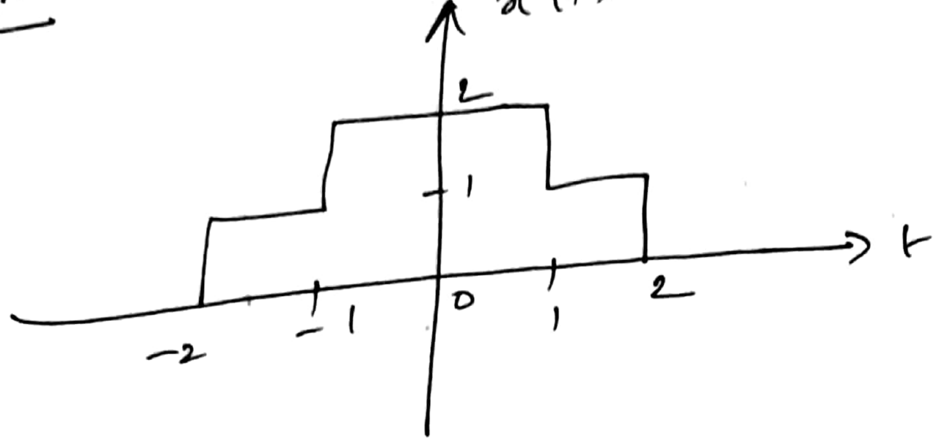
$$\boxed{y(t) = x(at)}$$

* If $a>1$, the signal $y(t)$ is a <u>compressed version</u> of $x(t)$

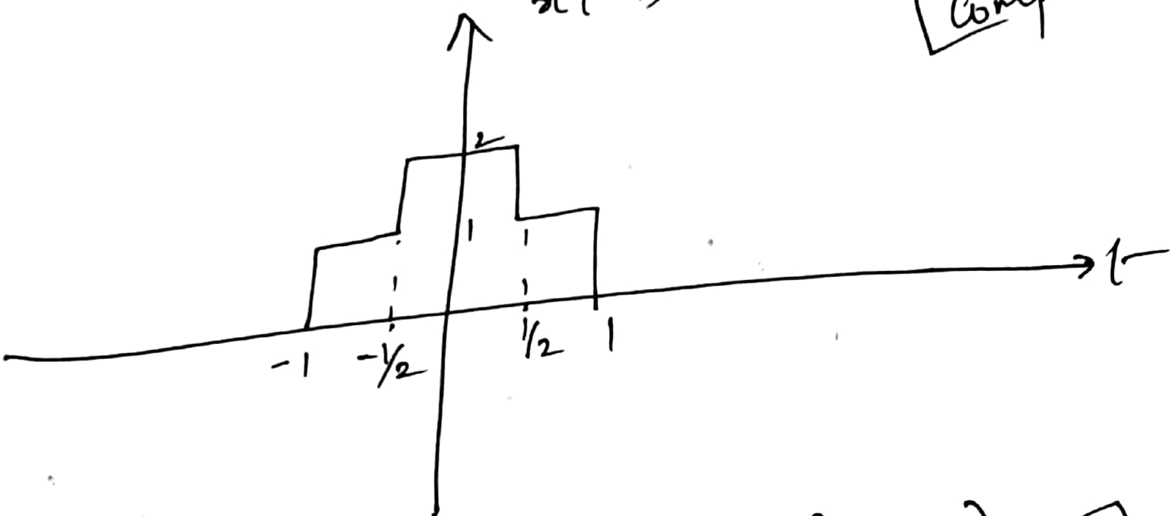
* If $0<a<1$, the signal $y(t)$ is <u>expanded</u> version of $x(t)$.

eg:-

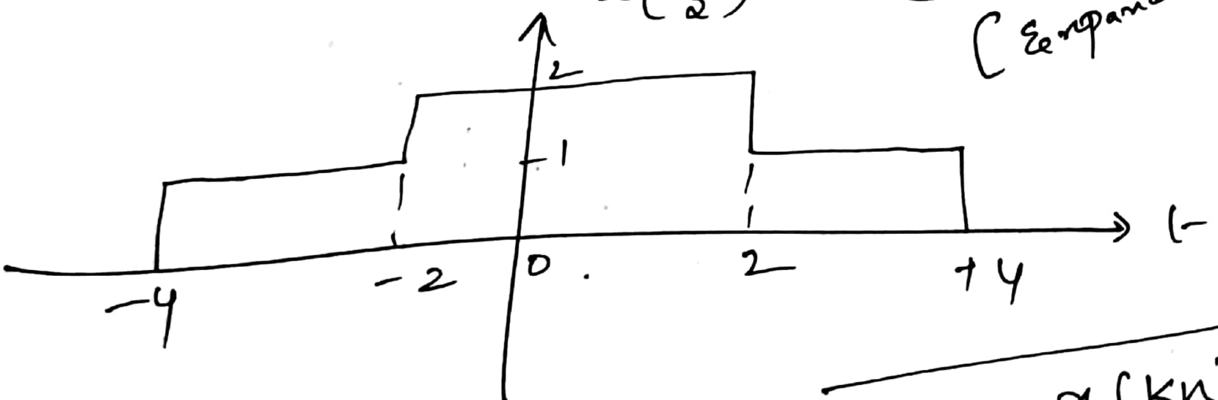
$x(t)$



$x(2t)$ $(a = 2)$ [Compressed]



$x\left(\frac{t}{2}\right)$ $(a = ½)$ [Expanded].



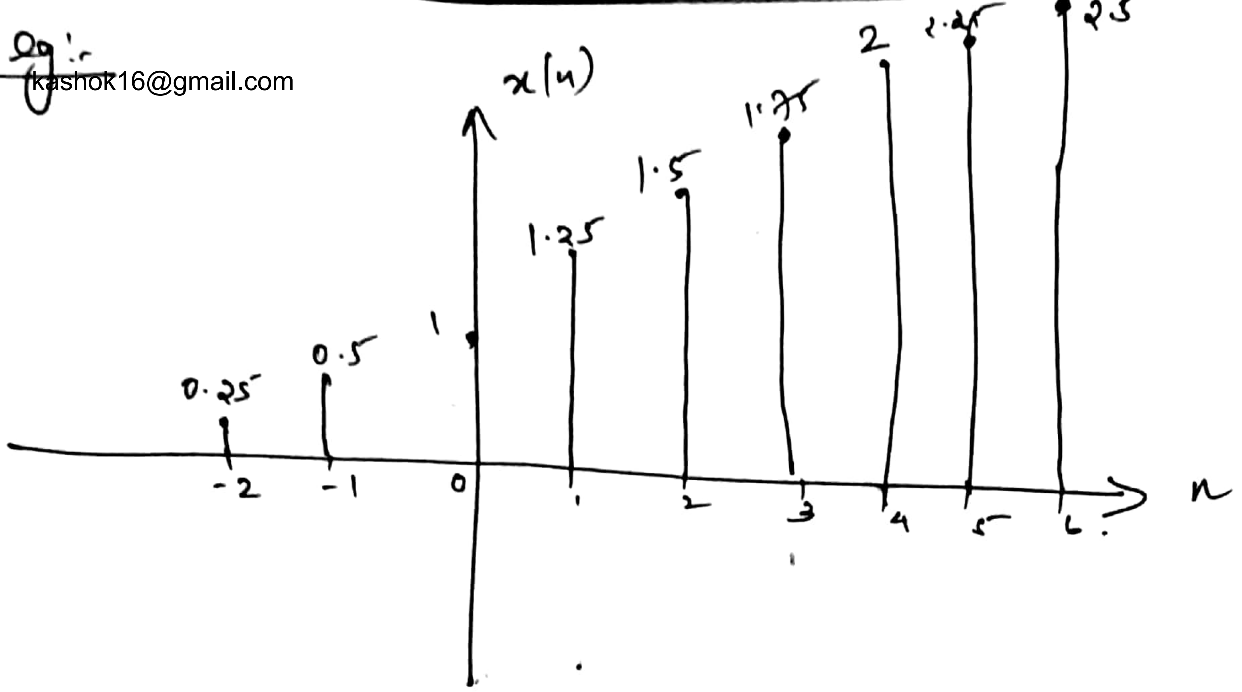
→ In the discrete-time sequence, $\boxed{y(n) = x(kn)}$; $k > 0$

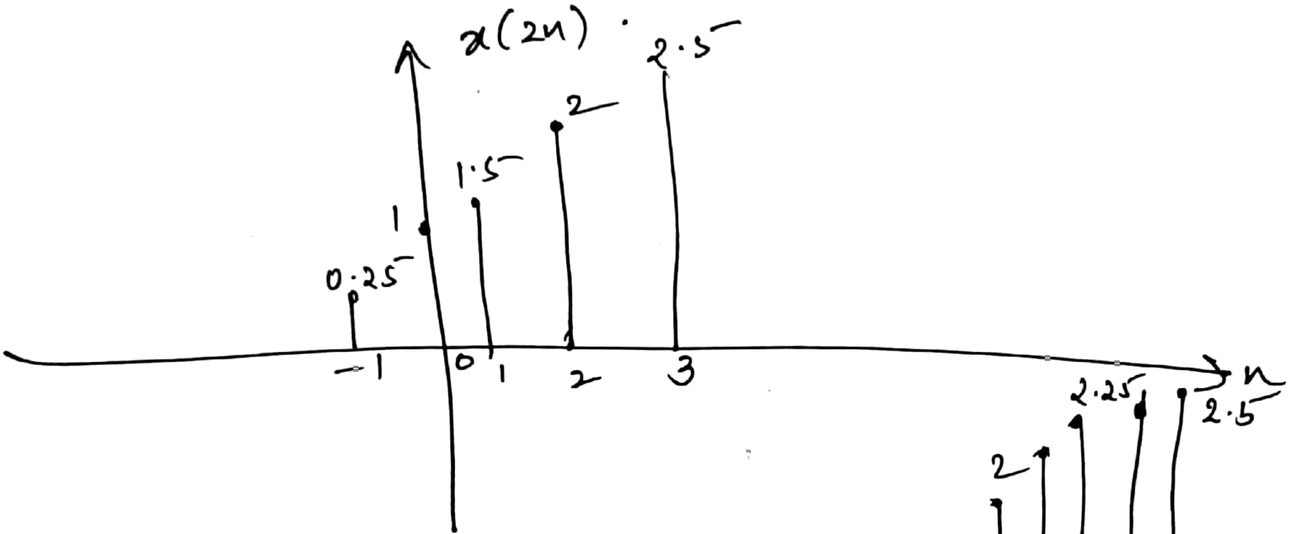
where, 'k' is an Integer.

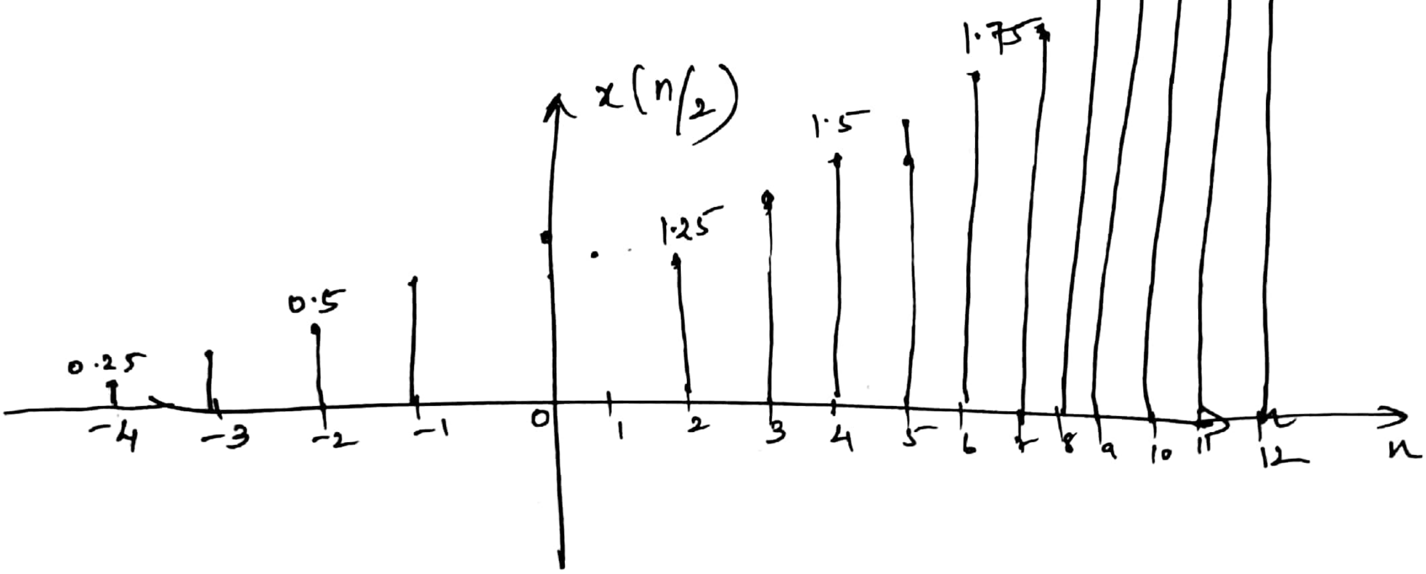
* If $k > 1$, the samples would be lost

If $k < 1$, additional samples will be taken.

eg:-


x(n)
0.25
0.5
1
1.25
1.5
1.75
2
2.25
2.5
-2
-1
0
1
2
3
4
5
6
n


x(2n)
0.25
1
1.5
2
2.5
-1
0
1
2
3
n

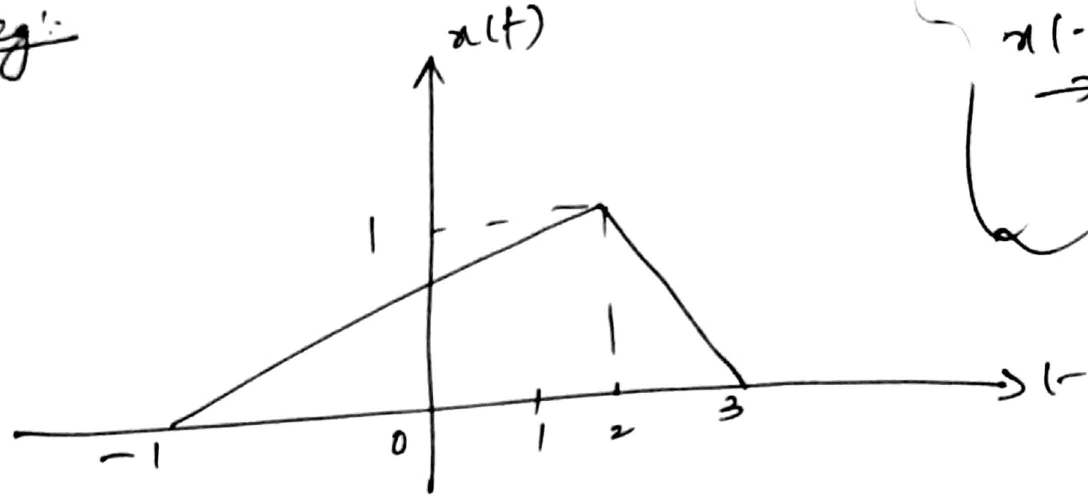

x(n/2)
0.25
0.5
1.25
1.5
1.75
2
2.25
2.5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
12
n

(ii) **Time Shifting:-** Let $x(t)$ be a continuous-time signal. Then the signal $y(t) = x(t-t_0)$ is known as the time shift. Shifted version of $x(t)$, where 't_0' is the time shift of the signal is shifted.

* If $t_0 > 0$, the waveform of the signal is shifted to the right. If $t_0 < 0$, the waveform is shifted to the left.

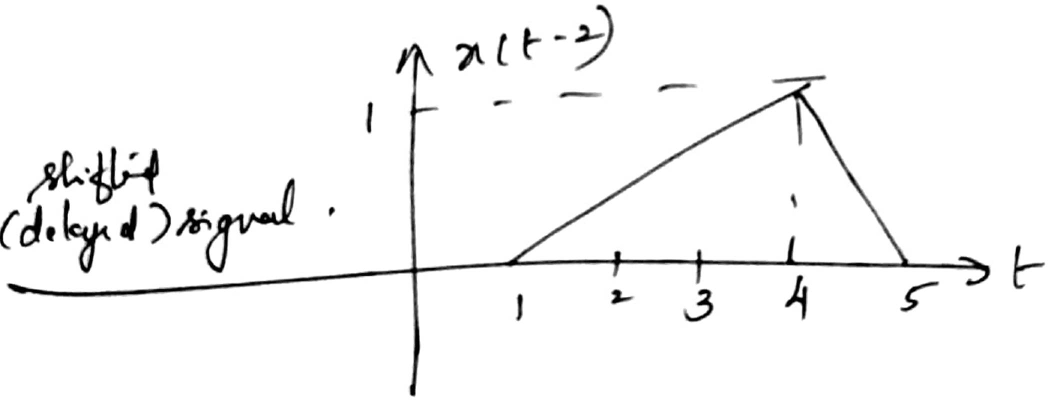
Note
$x(-t+2)$ - Right-shift
$x(-t-2)$ → left-shift

eg:-

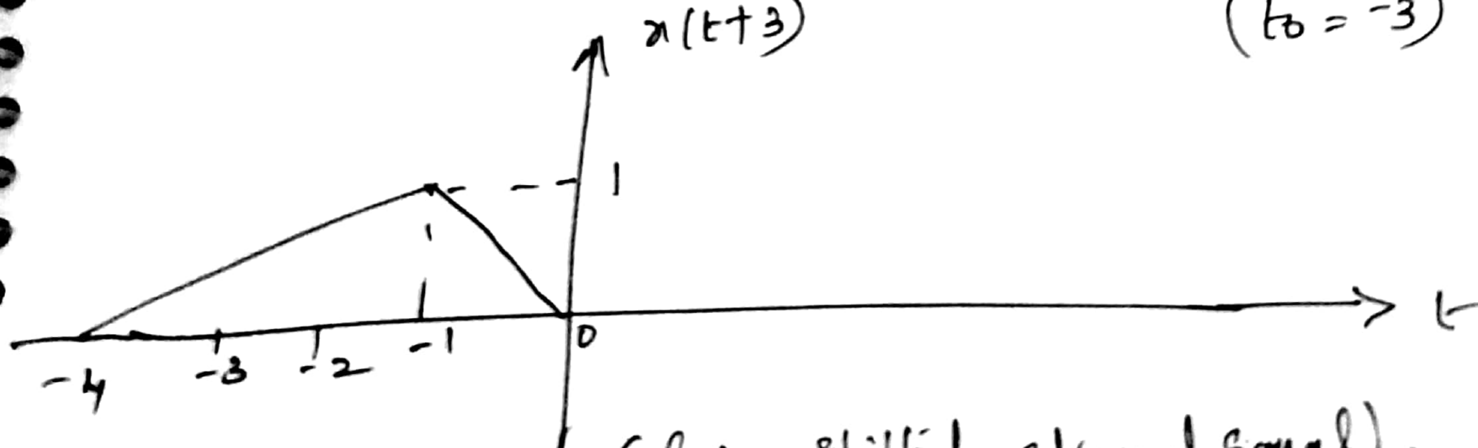


$(t_0 = 2)$

Right shifted (delayed) signal.



$(t_0 = -3)$

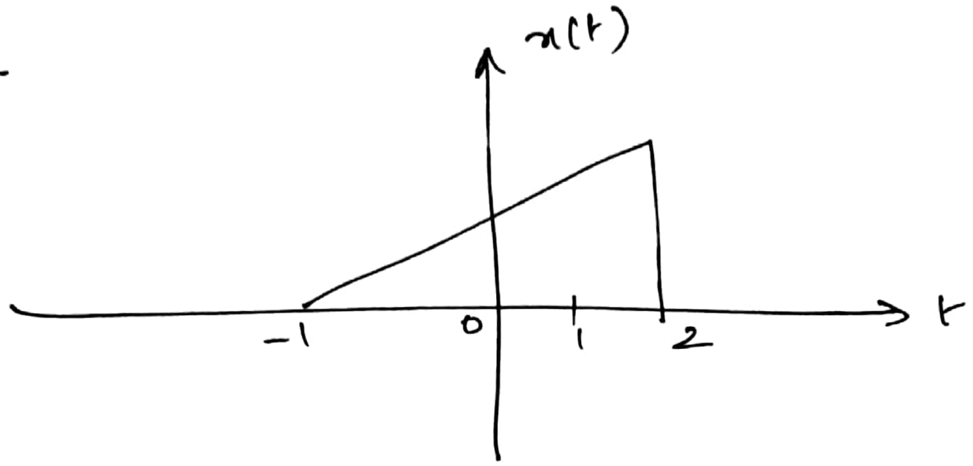


(left-shifted advanced signal).

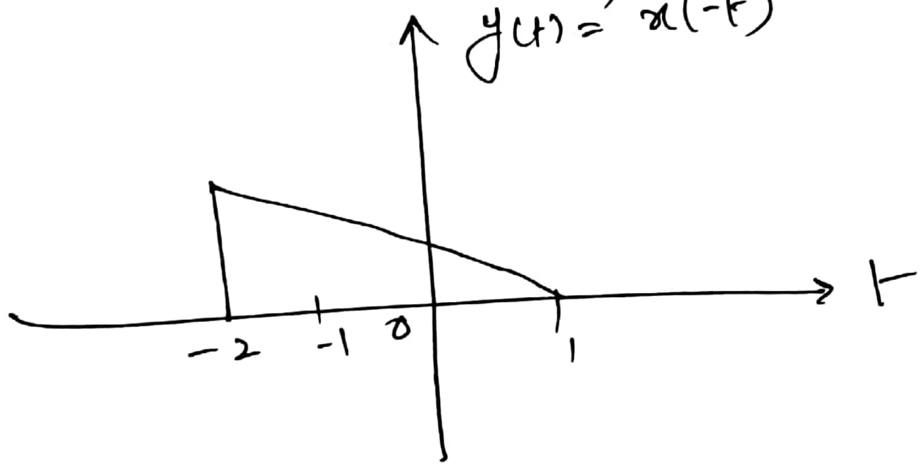
(iv) Time Reversal / Reflection

Let $x(t)$ be a continuous-time signal. Then the signal $y(t) = x(-t)$ is known as the reflected version of $x(t)$ about the amplitude axis.

eg:-



Reflected signal of $x(t)$ is shown as:-



Precedence Rule :-

If a signal operation consists of all operations like time reversal, shifting, scaling & amplitude scaling following order is used to perform the entire operation.

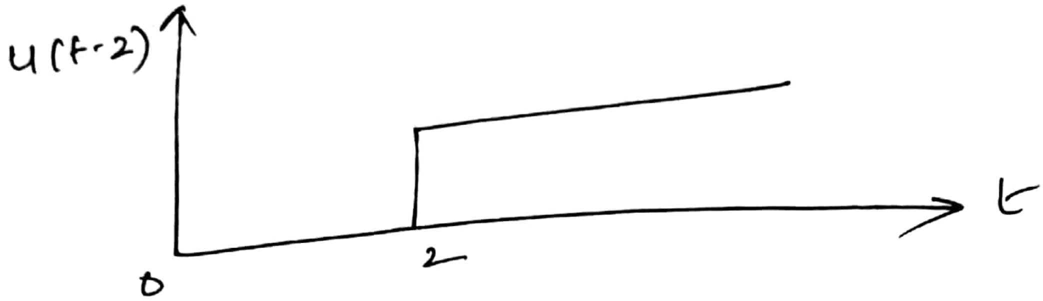
1. Time Reversal.
2. Time shifting.
3. Time scaling.
4. Amplitude scaling.

Q: 35. Sketch the following signal.

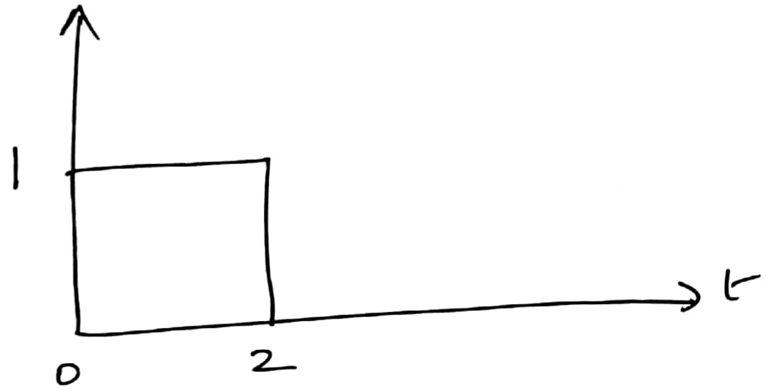
1.1

$$x(t) = u(t) - u(t-2).$$

Sol



$x(t) = u(t) - u(t-2)$

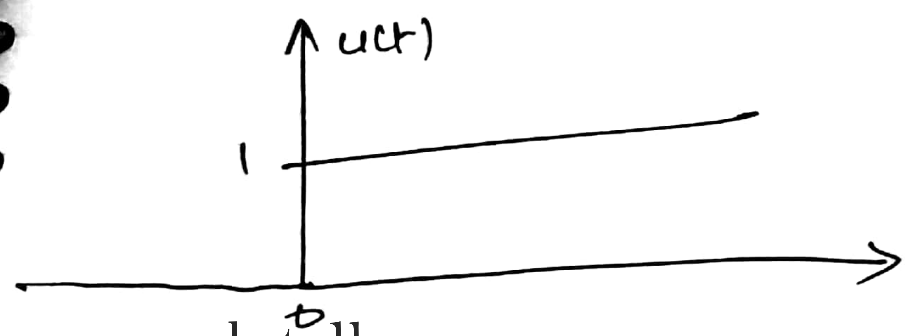


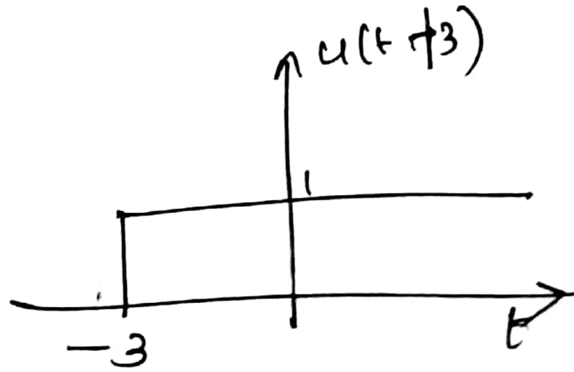
1.2

Q: 36 Sketch the signal

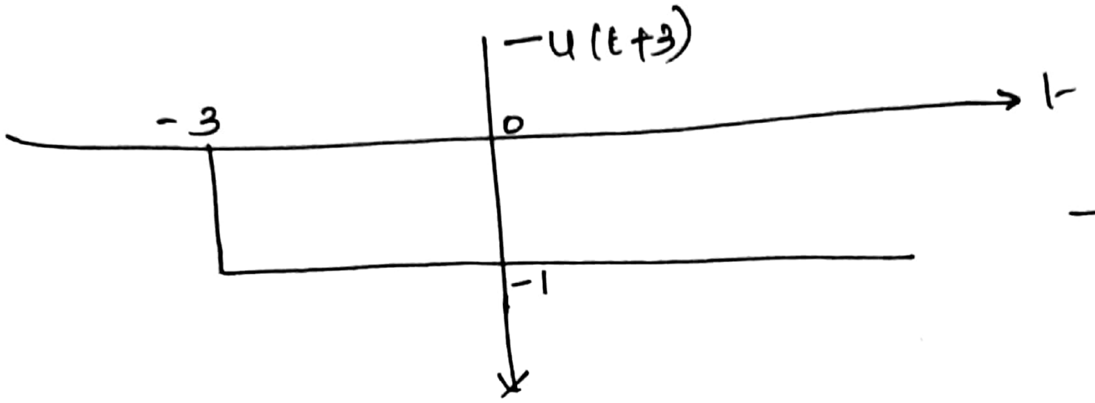
$$x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3).$$

Solⁿ * To plot $-u(t+3) \Rightarrow x_1(t)$.



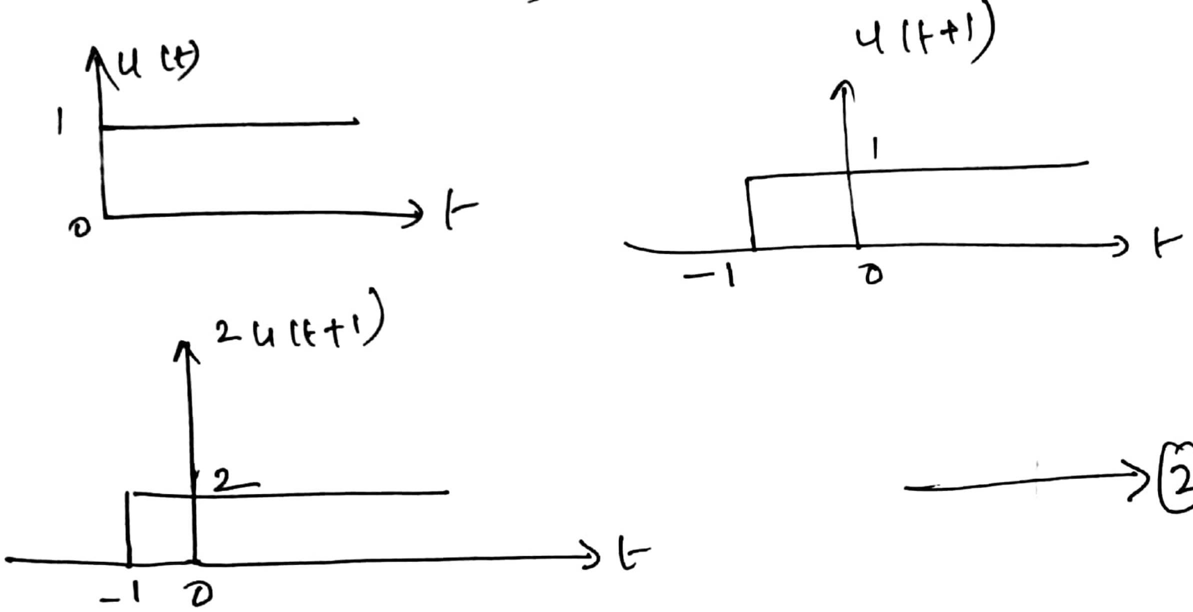


$-u(t+3) \Rightarrow$ Amplitude is Multiplied by -1



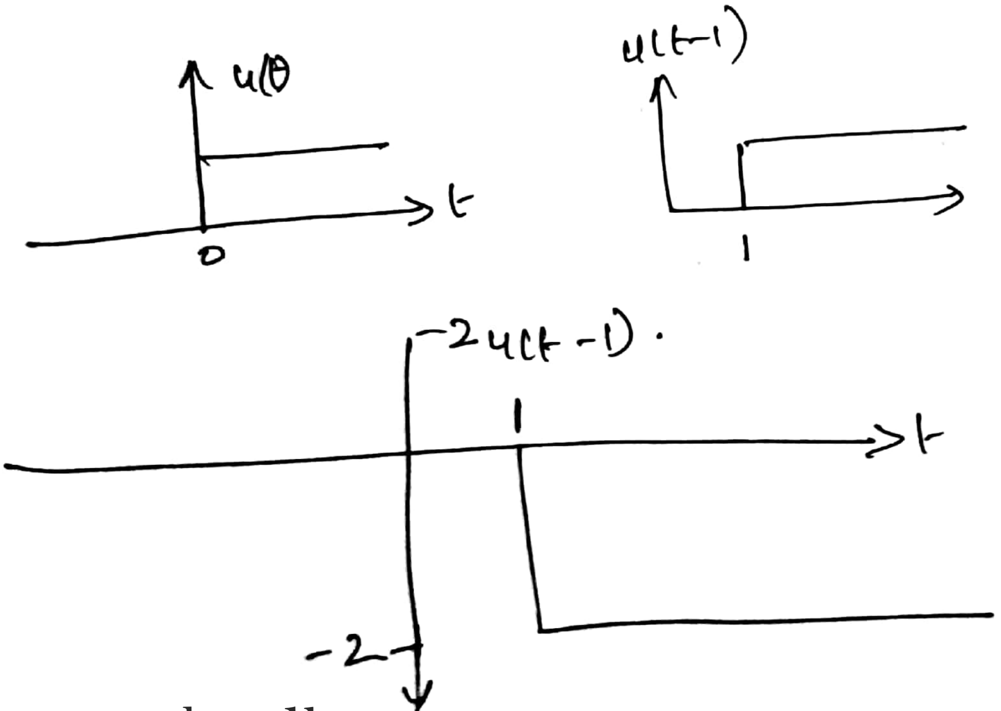
$\rightarrow$ (1)

* Plot $2u(t+1)$



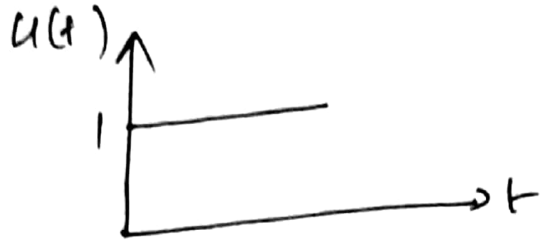
$\rightarrow$ (2)

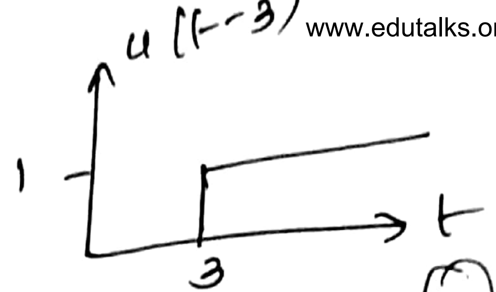
* plot $-2u(t-1)$



$\rightarrow$ (3)

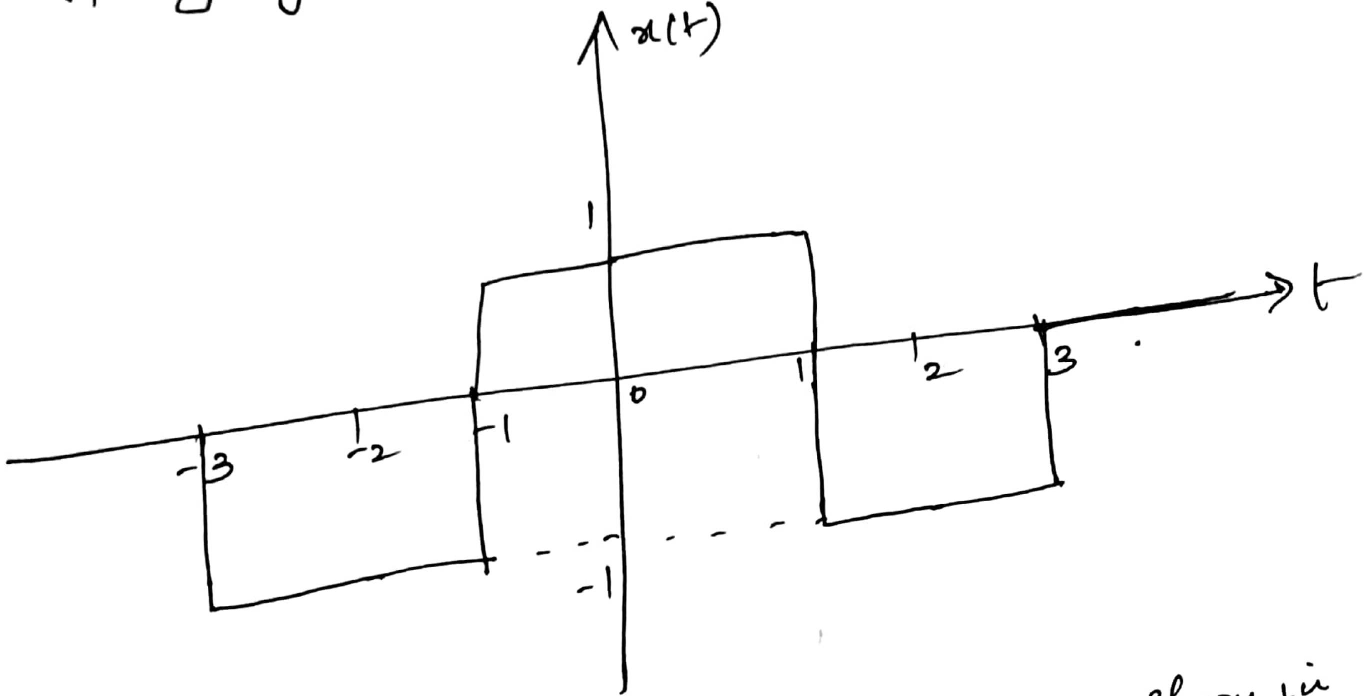
* To plot $u(t-3)$





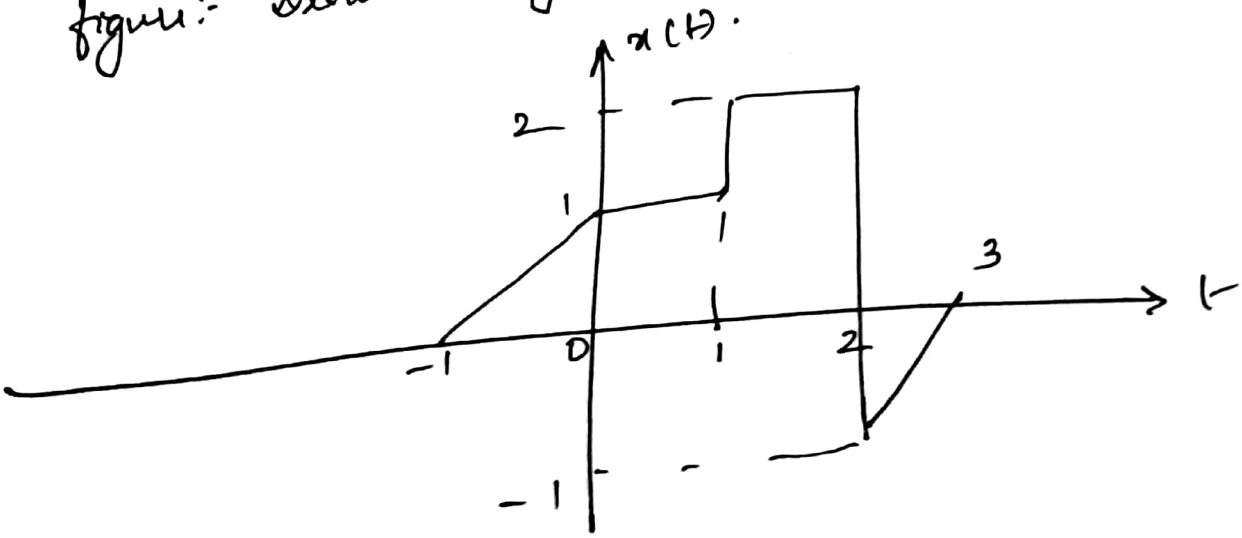
→ (4).

Adding fig (1) + (2) + (3) + (4).

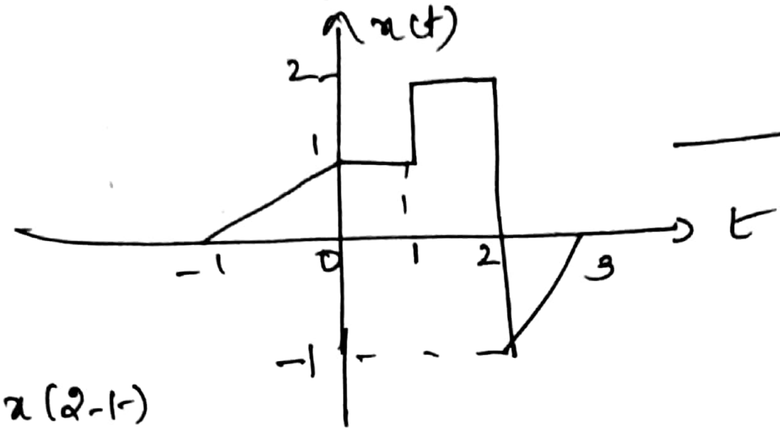


1.4

Q: 3.7 A continuous-time signal $x(t)$ is shown in figure:- Draw the signal. $y(t) = [x(t) + x(2-t)]\, u(1-t)$



Sol^n — We have the given signal.

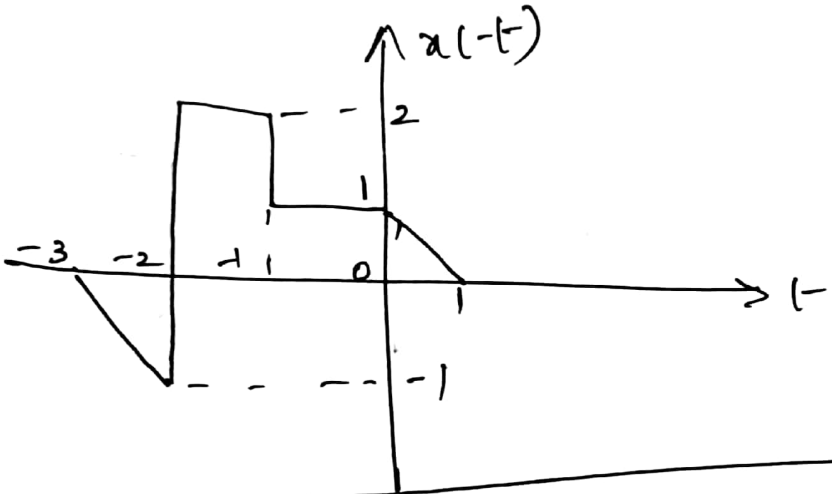


$\rightarrow$ (1)

To draw $x(2-t)$

$\Rightarrow x(-t+2)$

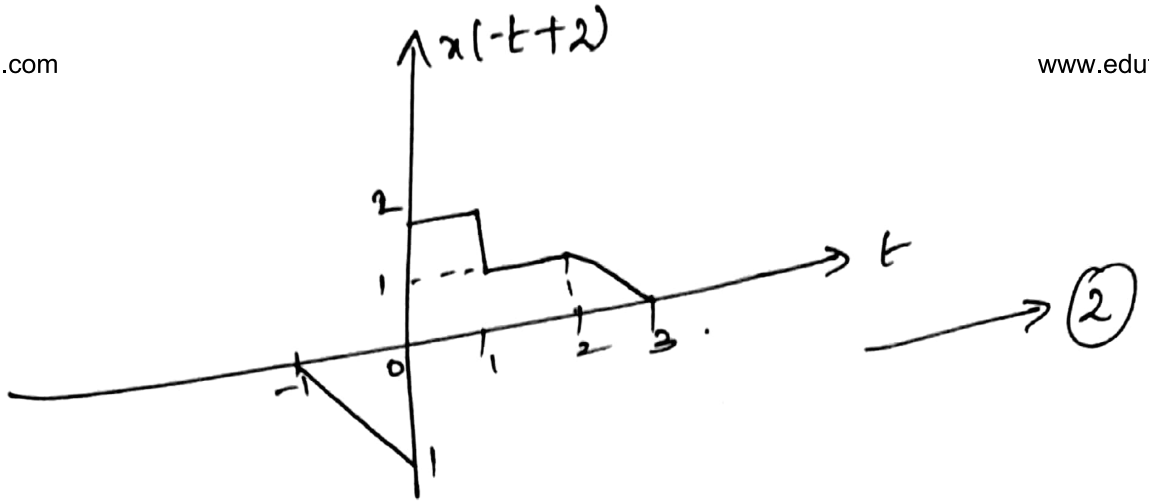
Perform Time Reversal and then time shifting.



Note

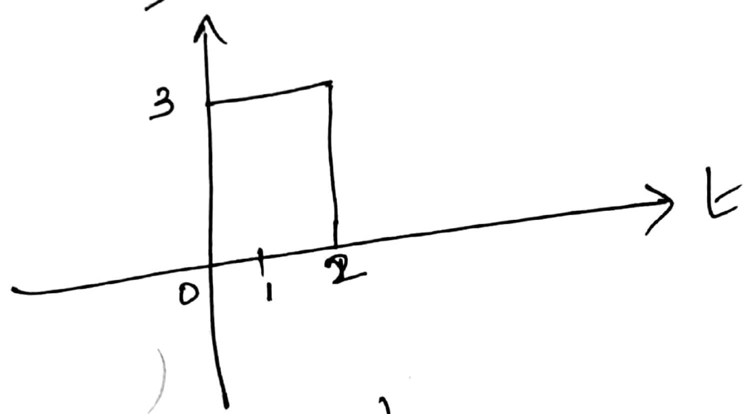
$$\boxed{\begin{array}{l} x(-t+t_0) \Rightarrow \text{Right shift} \\ x(-t-t_0) \Rightarrow \text{Left shift} \end{array}}$$ *

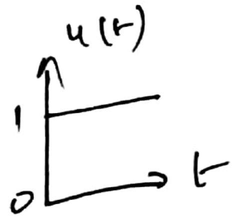
$$\boxed{\begin{array}{l} x(t-t_0) \Rightarrow \text{Right shift} \\ x(t+t_0) \Rightarrow \text{Left shift.} \end{array}}$$

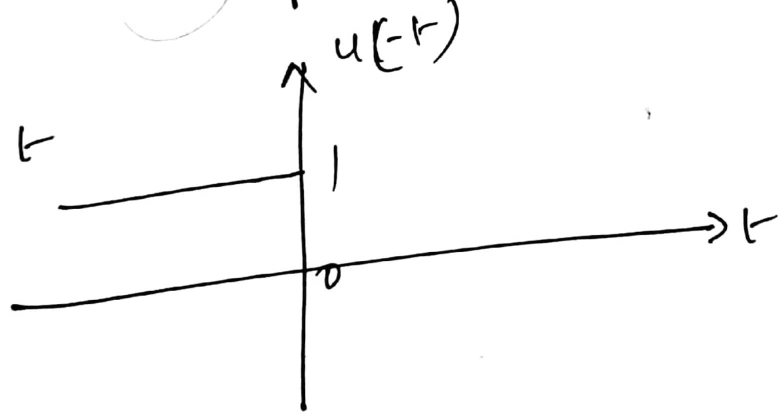


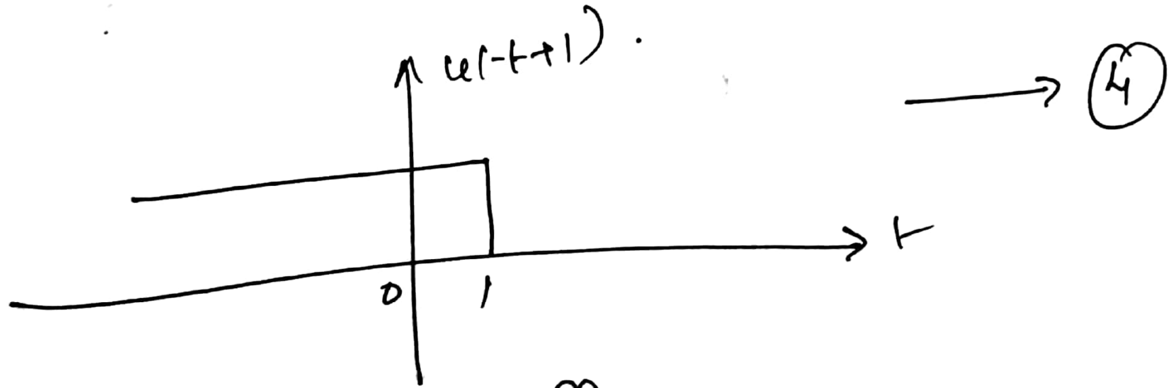
Add fig ① & ②

[x(t) + x(2-t)] → ③

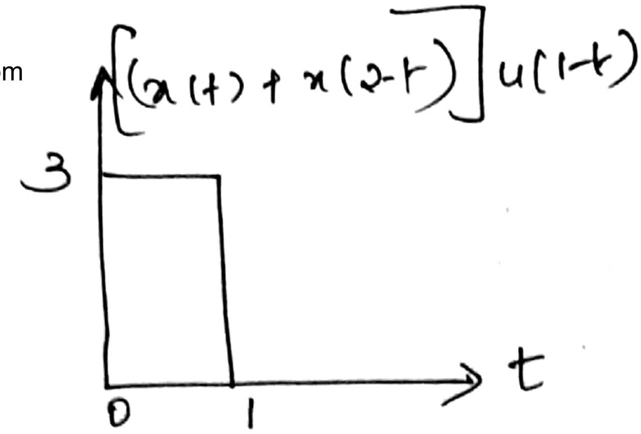






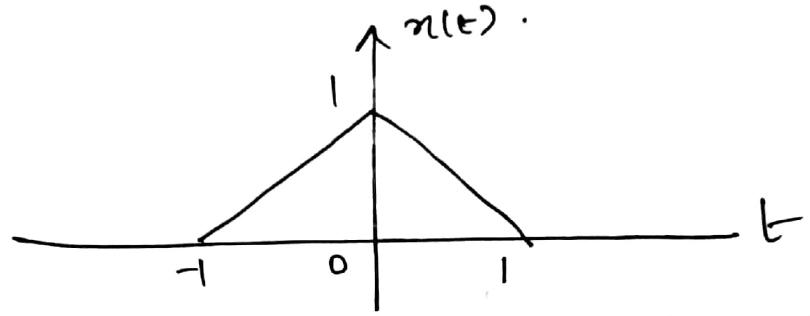


Multiply fig ③ × fig ④.



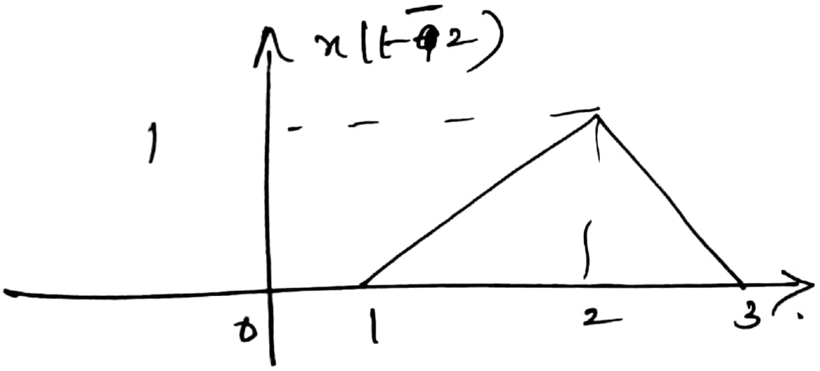
$\boxed{Q: 38}$ For a continuous time-signal $x(t)$ shown in figure, sketch the signal $y(t) = x(3t-2)$

Given :-

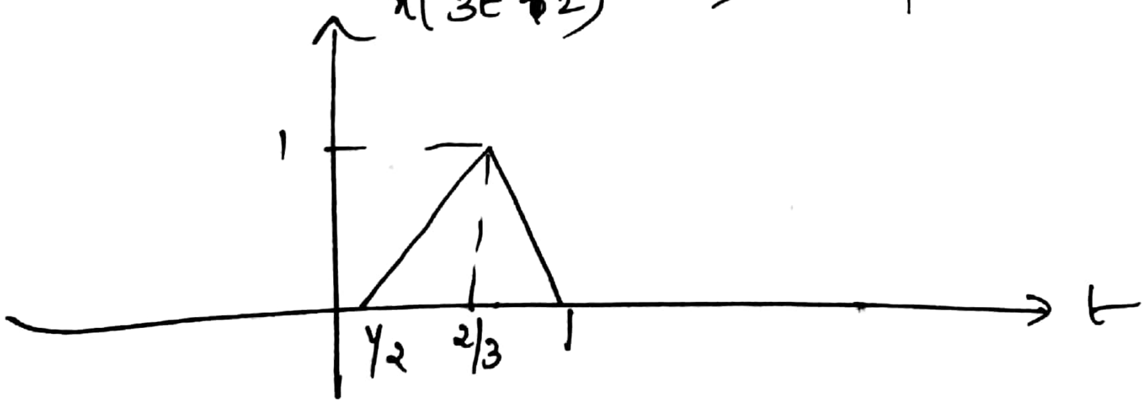


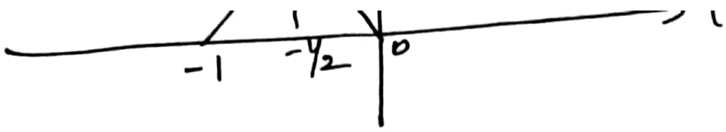
Soln

First perform time shifting and then time scaling.



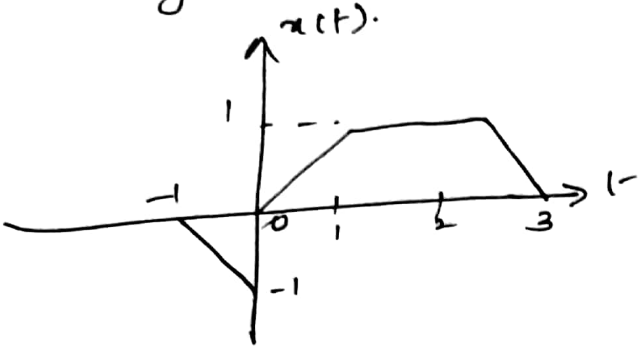
$x(3t-2) \Rightarrow$ Compress the signal by $\frac{1}{3}$rd.

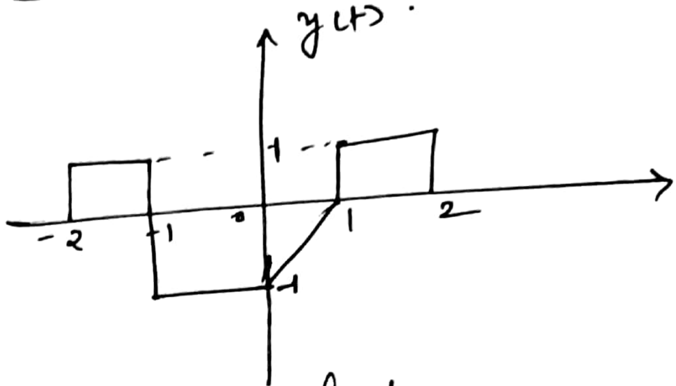




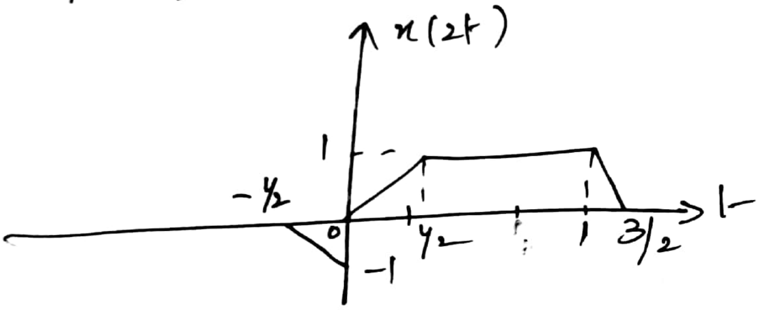
1.10

$\boxed{Q:40}$. Two continuous time signals $x(t)$ and $y(t)$ are given below: Draw $\boxed{z(t) = x(2t)\cdot y(2t+1)}$.



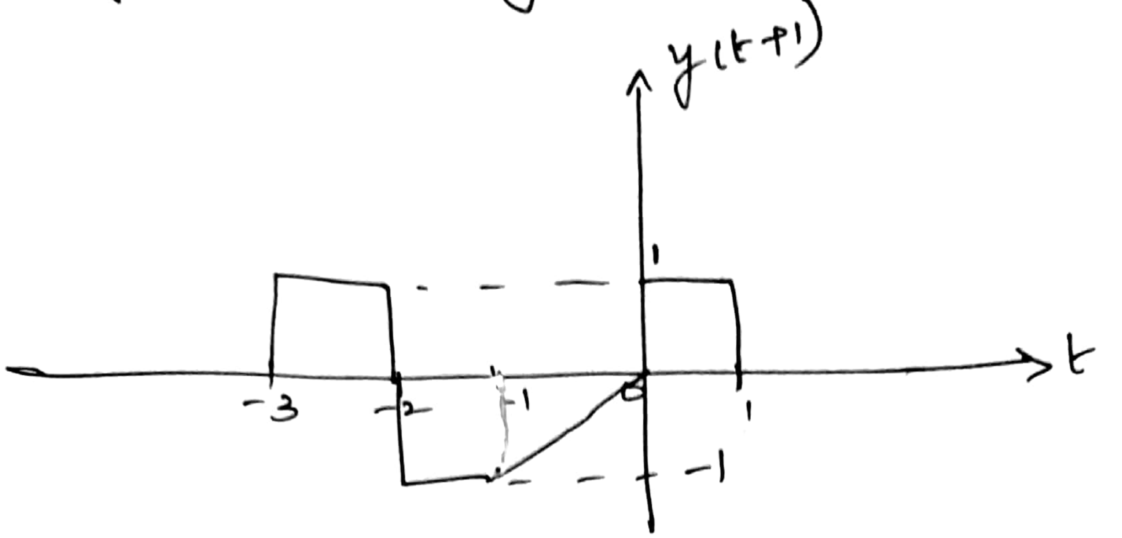


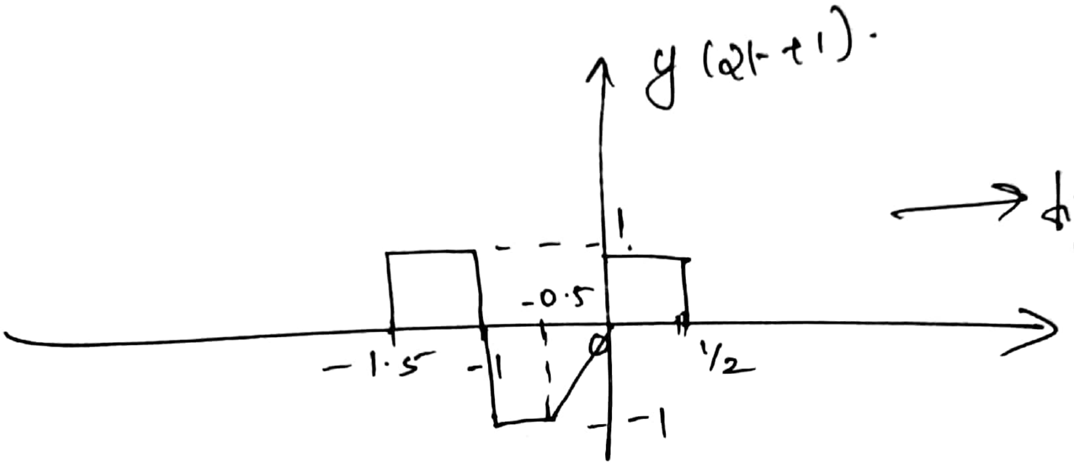
Sol^n To draw $x(2t)$ - compress by half.



$\longrightarrow$ fig (1)

* To draw $y(2t+1)$, plot $y(t+1)$ first & perform time scaling.



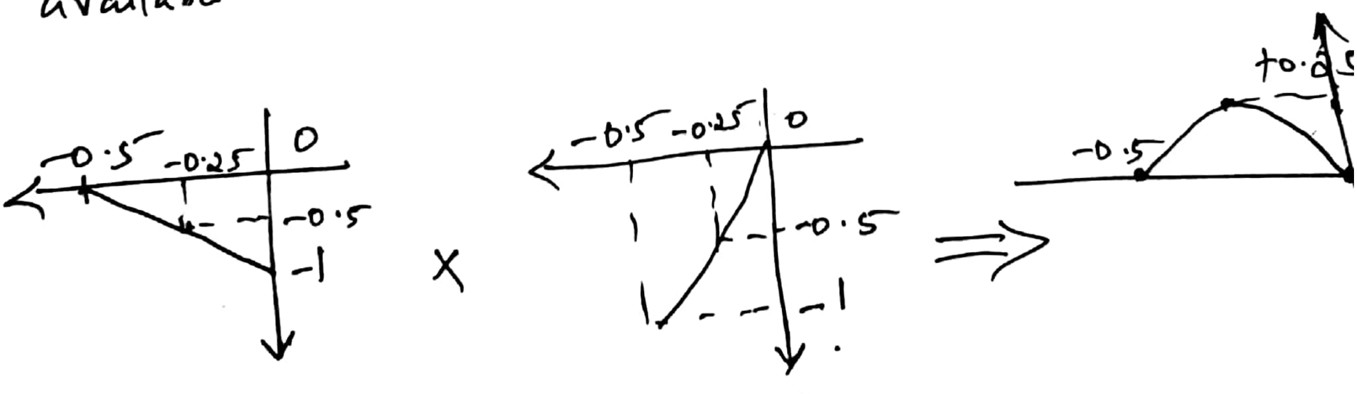


→ fig ②

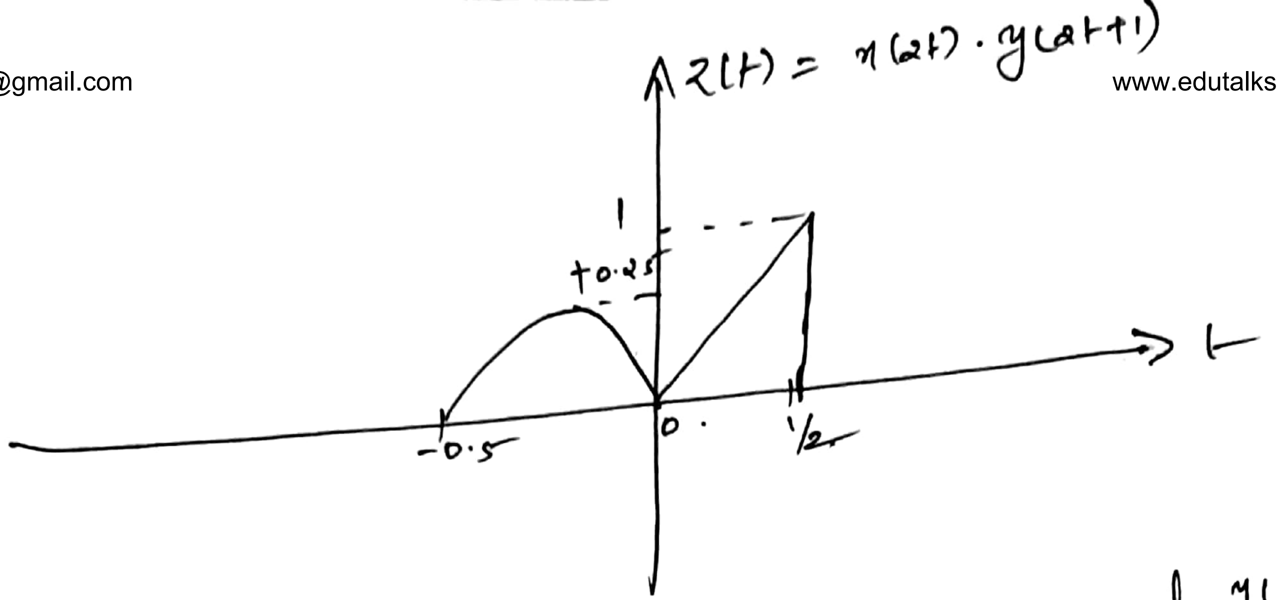
Multiply fig ① & fig ②.

* Till $-1.5 \le t \le -0.5$ $x(2t)$ is zero
∴ $z(t)$ is also zero.

* During $-0.5 \le t \le 0$ Both $x(2t)$ & $y(2t+1)$ are available.

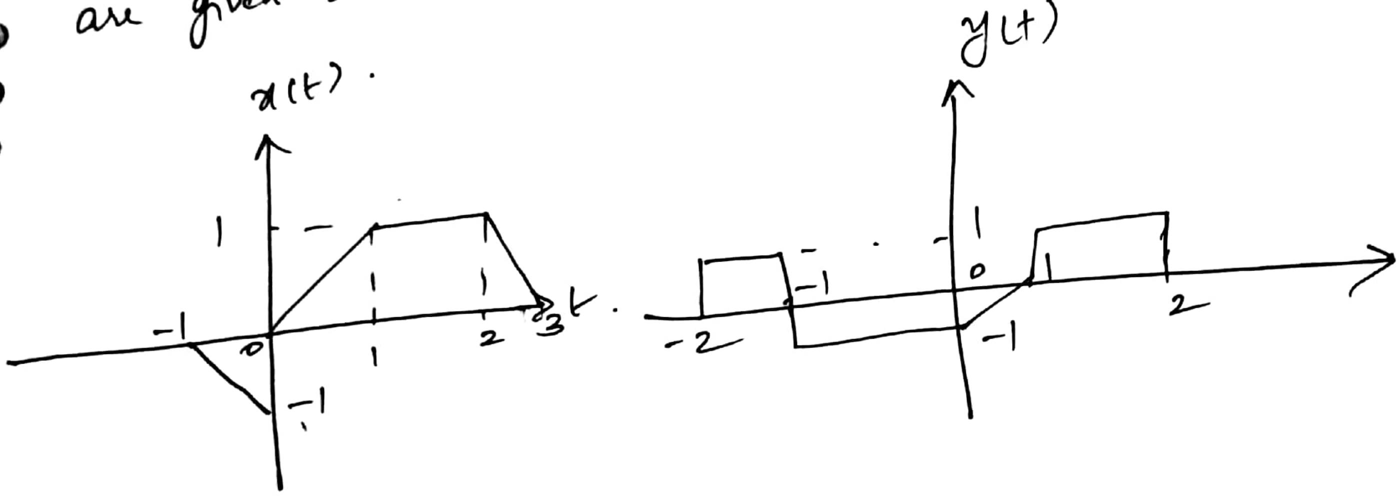


Multiply.



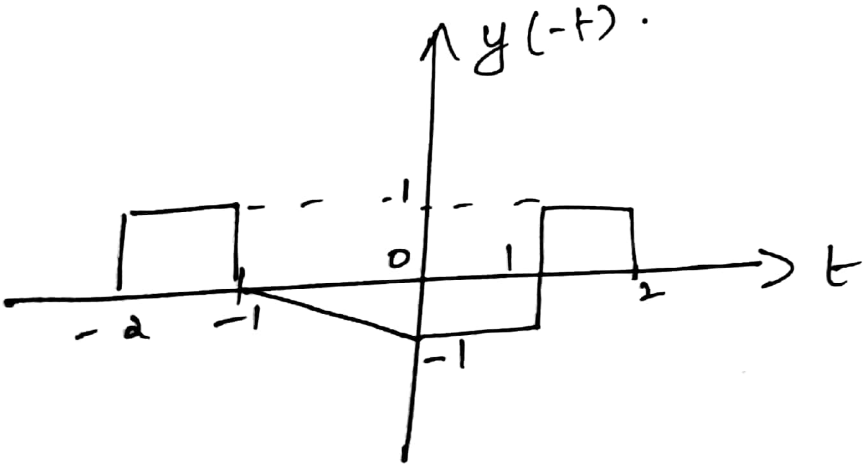
1.11

Q: (i) Two continuous time signals x(t) and y(t) are given below. Draw $\boxed{z(t) = x(t) \cdot y(-t-1)}$

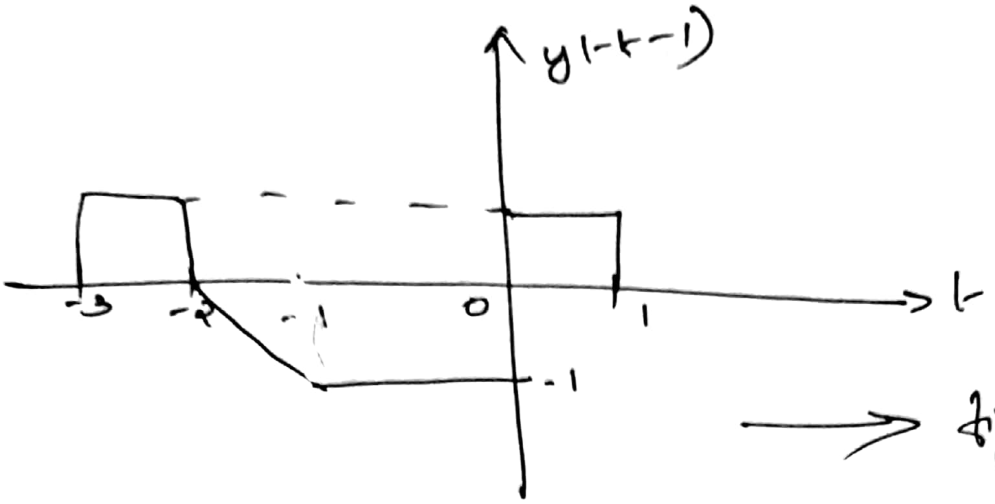


Sol^n To draw y(-t-1)

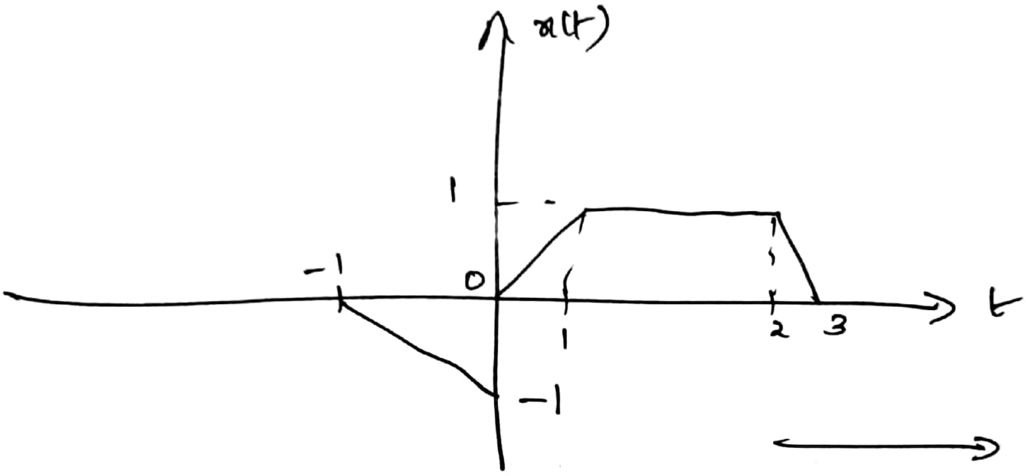
first draw y(-t) & then draw y(-t-1)



$y(-t-1) \rightarrow$ left shift

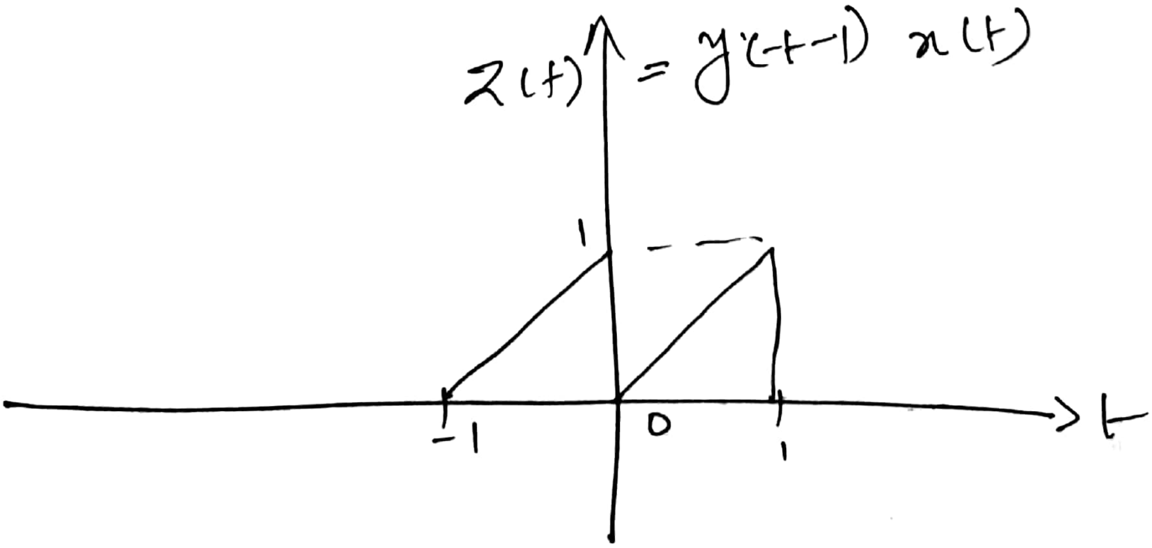


$\longrightarrow$ fig ①



$\longrightarrow$ fig ②

Multiply fig ① & fig ②



1.13

Q: 44

A discrete time signal $x(n)$ is shown in figure.

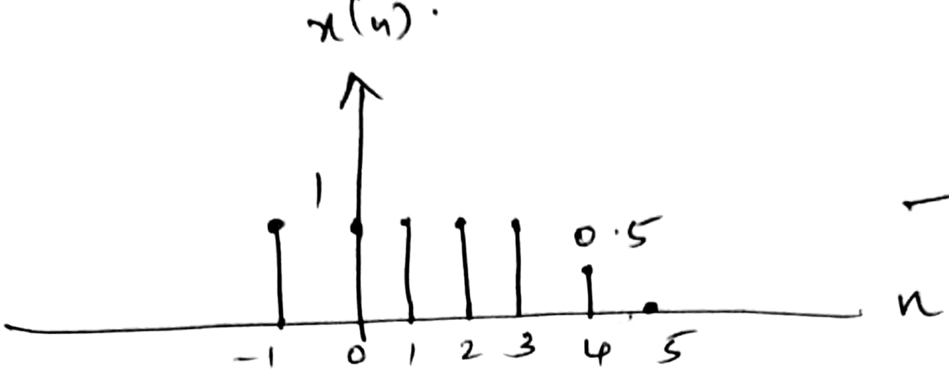


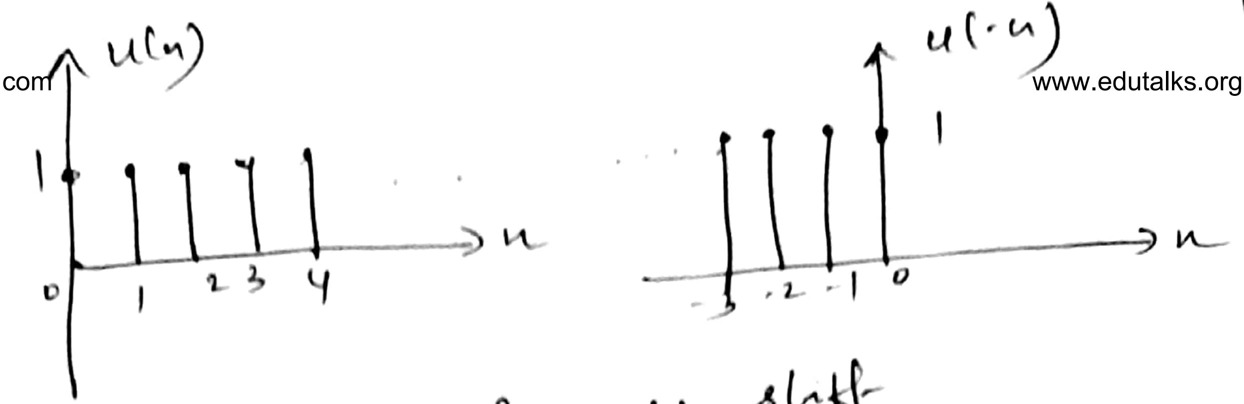
fig (1)

Sketch the signal $y(n) = x(n)\, u(2-n)$

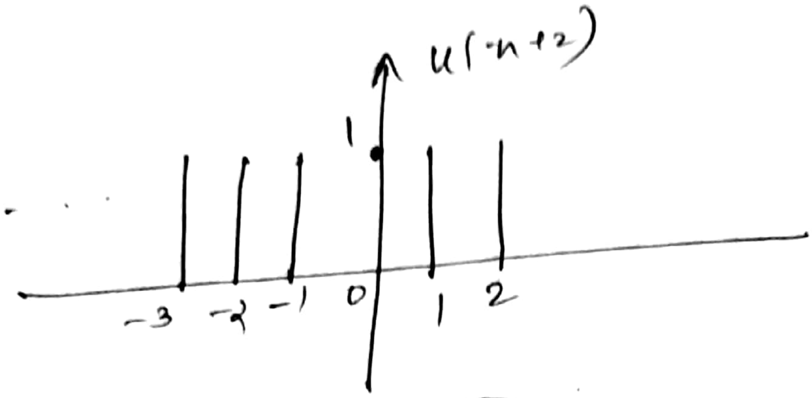
Sol

To draw $u(2-n) = u(-n+2)$

Do, Time ~~Scaling~~ Reversal and then Perform shifting.

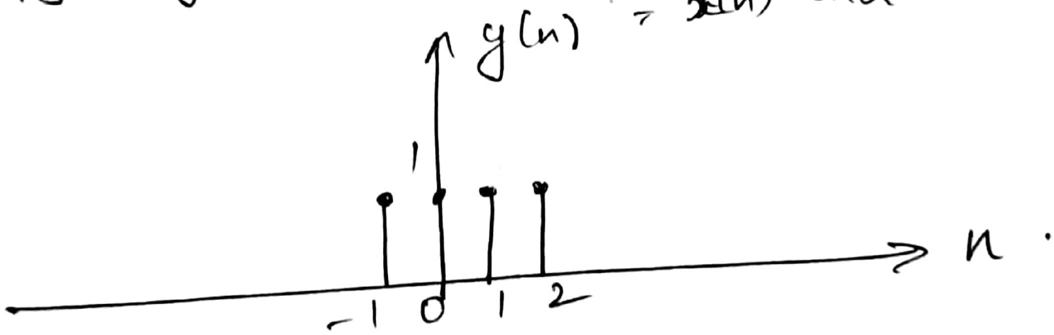


To draw $u(-n+2)$, do right shift



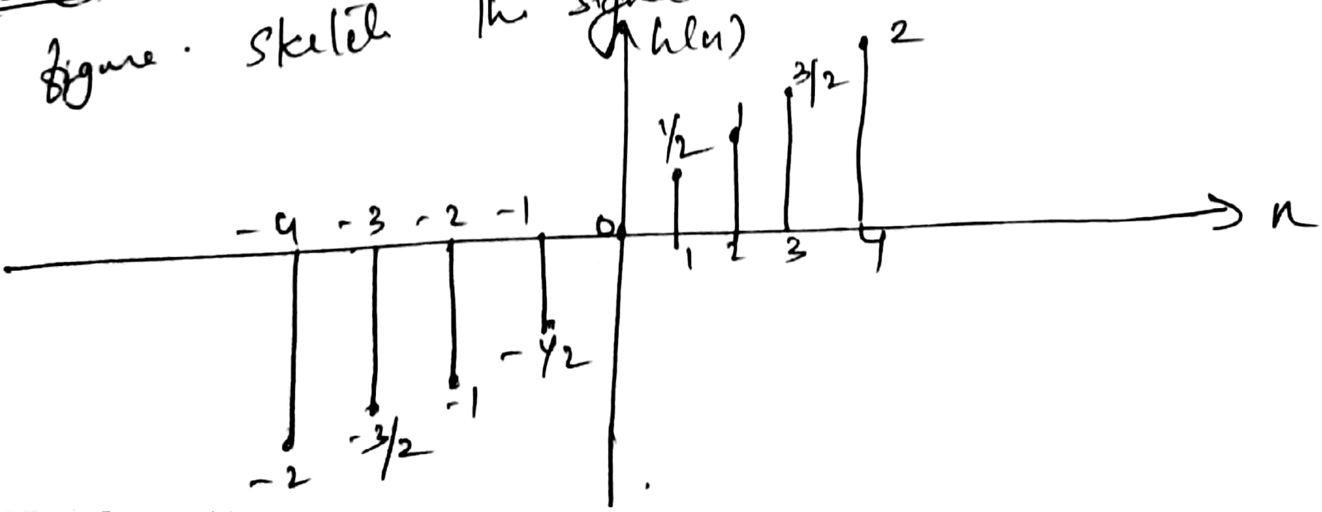
— fig (2)

Multiply fig (1) & fig (2)

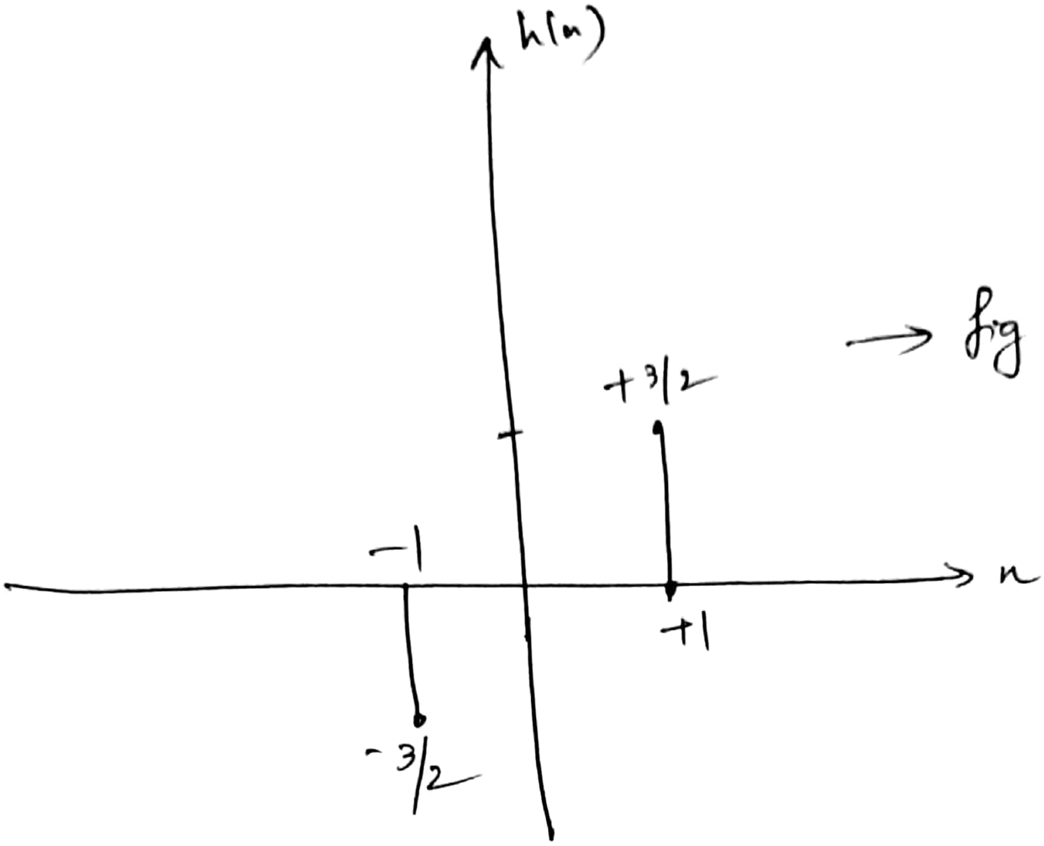


1.14

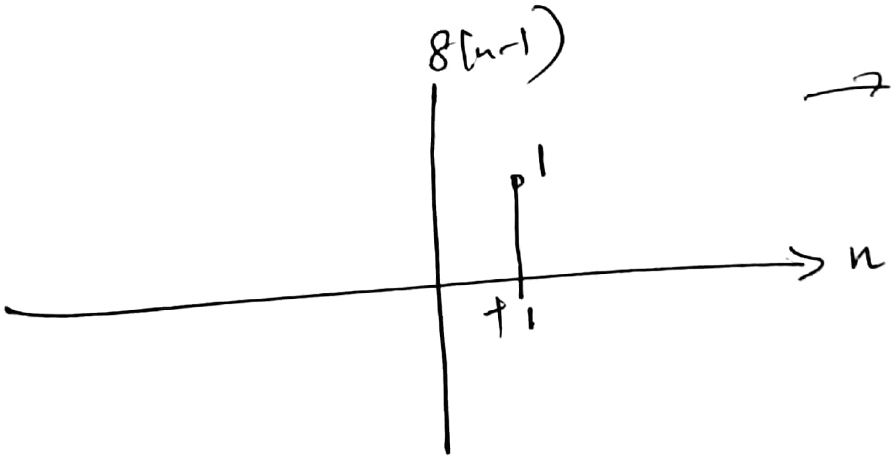
Q.45 A discrete time sequence $h(n)$ is shown in figure. Sketch the signal $x(n) = h(3n) \cdot \delta(n-1)$



Sol^n To draw $h(3n)$ remove the samples & compress by $\frac{1}{3}$rd.



→ fig ①



→ fig ②

Multiplying ① & ②