

4. RECTIFIERS:

4.1 Introduction

A **diode** generally refers to a two-terminal solid-state semiconductor device that presents a low impedance to current flow in one direction and a high impedance to current flow in the opposite direction. These properties allow the diode to be used as a one-way current valve in electronic circuits. *Rectifiers are a class of circuits whose purpose is to convert ac waveforms (usually sinusoidal and with zero average value) into a waveform that has a significant non-zero average value (dc component).* Simply stated, rectifiers are ac-to-dc energy converter circuits. Most rectifier circuits employ diodes as the principal elements in the energy conversion process; thus the almost inseparable notions of diodes and rectifiers.

This section discusses some simple **uncontrolled rectifier** circuits that are commonly encountered. The term *uncontrolled* refers to the absence of any control signal necessary to operate the primary switching elements (diodes) in the rectifier circuit. (The discussion of controlled rectifier circuits, and the controlled switches themselves, is more appropriate in the context of power electronics applications). Rectifiers are the fundamental building block in dc power supplies of all types and in dc power transmission used by some electric utilities.

Small signal diodes can be used as rectifiers in low-power, low current (less than 1-amp) rectifiers or applications, but where larger forward bias currents or higher reverse bias blocking voltages are involved the PN junction of a small signal diode would eventually overheat and melt so larger more robust **Power Diodes** are used instead. The power semiconductor diode, known simply as the **Power Diode**, has a much larger PN junction area compared to its smaller signal diode cousin, resulting in a high forward current capability of up to several hundred amps (KA) and a reverse blocking voltage of up to several thousand volts (KV). Since the power diode has a large PN junction, it is not suitable for high frequency applications above 1MHz, but special and expensive high frequency, high current diodes are available. For high frequency rectifier applications **Schottky Diodes** are generally used because of their short reverse recovery time and low voltage drop in their forward bias condition.

There are two types of rectifiers

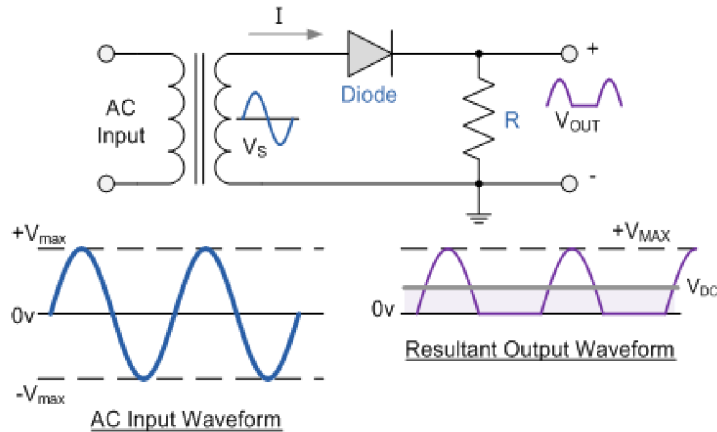
- Half Wave (HW) rectifier
- Full Wave (FW) rectifier

4.2 Half wave rectifier

The diode in a half wave rectifier circuit passes just one half of each complete sine wave of the AC supply in order to convert it into a DC supply. Then this type of circuit is called a "half-wave" rectifier because it passes only half of the incoming AC power supply as shown below.

During each "positive" half cycle of the AC sine wave, the diode is *forward biased* as the anode is positive with respect to the cathode resulting in current flowing through the diode. Since the DC load is resistive (resistor, R), the current flowing in the load resistor is therefore proportional to the voltage (*Ohm's Law*), and the voltage across the load resistor will therefore be the same as the supply voltage, V_s (minus V_f), that is the "DC" voltage across the load is sinusoidal for the first half cycle only so $V_{out} = V_s$.

During each "negative" half cycle of the AC sine wave, the diode is *reverse biased* as the anode is negative with respect to the cathode therefore, No current flows through the diode or circuit. Then in the negative half cycle of the supply, no current flows in the load resistor as no voltage appears across it so $V_{out} = 0$.



Disadvantages of HWR

- Low output because one half cycle only delivers output
- A.C. component more in the output
- Requires heavy filter circuits to smooth out the output

Peak inverse Voltage

In the rectifier circuit given above, during the negative half cycle of the secondary voltage, the diode is reverse biased. As there is no voltage across the load R_L during this half cycle, the whole secondary voltage will come across the diode. When the secondary voltage reaches its maximum V_m , in the negative half cycle the voltage across the diode is also maximum. This maximum voltage is known as **peak inverse voltage (PIV)**. It is the maximum voltage the diode must withstand during the reverse bias half cycle of the input. In the case of HWR, $PIV = V_m$.

4.2.1 Performance of half wave rectifier

a. Peak inverse Voltage

In the case of HWR, $PIV = V_m$.

b. D.C output

The DC output voltage or current of the rectifier is the average value of the output voltage or current.

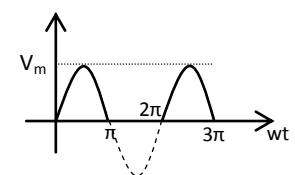
i.e
$$I_{dc} = I_{av} = \frac{\text{Area under one complete cycle}}{\text{Time period}} \tag{4.1}$$

We know that the average value of a sine wave over a complete cycle is zero because it is symmetrical. But in case of a half wave rectifier output, it consists only alternate half (+ ve or -ve) cycles, hence, there will be some dc current flowing through R_L .

Let, $v_s = V_m \sin \omega t$ be the input voltage, $i_L = I_m \sin \omega t$ be the current flowing through the load resistor R_L and r_d be forward resistance of the diode.

Mathematically the output wave form can be described as,

$$i_L = \begin{cases} I_m \sin \omega t & : 0 \leq \omega t \leq \pi \\ 0 & : \pi \leq \omega t \leq 2\pi \end{cases} \tag{4.2}$$



Here area = $\int_0^{2\pi} i_L d(\omega t)$
 $= \int_0^{\pi} I_m \sin \omega t d(\omega t) + \int_{\pi}^{2\pi} I_m \sin \omega t d(\omega t)$

$$= \int_0^\pi I_m \sin \omega t d(\omega t) + 0 \quad (\text{from (4.2)})$$

$$= I_m [-\cos \omega t]_0^\pi = I_m \quad 2I_m \quad (4.3)$$

$$\text{Time period} = 2\pi \quad (4.4)$$

$$\text{Now,} \quad I_{dc} = I_{av} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi} \quad (4.5)$$

$$\text{Voltage across } R_L, \quad V_{dc} = I_{dc} R_L = \frac{I_m R_L}{\pi} \quad (4.6)$$

$$\text{While considering the forward resistance } r_d, \quad I_m = \frac{V_m}{R_L + r_d} \quad (4.7)$$

$$\therefore V_{dc} = \frac{1}{\pi} \frac{V_m}{R_L + r_d} \quad I = \frac{1}{\pi} \frac{V_m}{(1 + \frac{r_d}{R_L})} \cong \frac{V_m}{\pi} \text{ since } r_d \ll R_L. \quad (4.8)$$

c. R.M.S. Value

The R.M.S. value is the effective value of the current flowing through the load and is given by

$$I_{rms} = \sqrt{\frac{\text{Area under the squared rectified wave over a cycle}}{\text{Time period}}} \quad (4.9)$$

In case of a half wave rectifier,

$$i_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_L^2 d(\omega t)} = i_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \omega t)^2 d(\omega t)} \quad (4.10)$$

$$= \sqrt{\frac{1}{2\pi} \int_0^\pi (I_m \sin \omega t)^2 d(\omega t) + \frac{1}{2\pi} \int_\pi^{2\pi} (I_m \sin \omega t)^2 d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_0^\pi (I_m \sin \omega t)^2 d(\omega t) + 0}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^\pi I_m^2 \sin^2(\omega t) d(\omega t)} = \sqrt{\frac{I_m^2}{2\pi} \int_0^\pi \left\{ \frac{1 - \cos 2\omega t}{2} \right\} d(\omega t)} = \sqrt{\frac{I_m^2}{4\pi} \left[\omega t - \frac{\sin \omega t}{2} \right]_0^\pi}$$

$$= \frac{I_m}{2} \quad (4.11)$$

$$\text{Also } V_{rms} = V_m / 2 \quad (4.12)$$

This is the rms value of the total load current which include d.c. value and a.c. components. In the out put of rectifier, the instantaneous value of a.c fluctuation is the difference of the instantaneous total value and the d.c. value.

i.e, the instantaneous a.c. value is given as

$$i' = i_L - I_{dc} \quad (4.13)$$

Here the rms value of the a.c component is given by,

$$i_{rms}' = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i_L - I_{dc})^2 d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i_L^2 - 2i_L I_{dc} + I_{dc}^2) d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_L^2 d(\omega t) - 2I_{dc} \frac{1}{2\pi} \int_0^{2\pi} i_L d(\omega t) + \frac{1}{2\pi} I_{dc}^2 \int_0^{2\pi} d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_L^2 d(\omega t) - I_{dc} I_{dc} + \frac{1}{2\pi} I_{dc}^2 \pi}$$

$$= \sqrt{I_{rms}^2 - I_{dc}^2} \quad (4.14)$$

d. Ripple factor(γ)

The purpose of the rectifier is to convert a.c. voltage to d.c., but no type of rectifier convert a.c. to perfect d.c. It produces pulsating d.c. This residual pulsation is called ripple.

The ripple factor indicates the effectiveness of a rectifier in converting a.c. to perfect d.c. It is the ratio of the ripple voltage to the d.c. voltage.

$$\text{i.e.} \quad \gamma = \frac{\text{Ripple Voltage}}{\text{D.C.Voltage}} \quad (4.15)$$

$$= \frac{\text{rms value of a.c.component}}{\text{Average or d.c.component}}$$

$$= \frac{V'_{rms}}{V_{dc}} \quad (4.16)$$

$$\text{or, } \gamma = \frac{I'_{rms}}{I_{dc}} \quad (4.18)$$

$$= \frac{\sqrt{I_{rms}^2 - I_{dc}^2}}{I_{dc}} = \sqrt{\frac{(I_m/2)^2}{(I_m/\pi)^2} - 1} = \sqrt{1.57 - 1} = 1.21 \quad (4.18)$$

From this value, it is clear that in HWR the a.c. component exceeds the d.c. component. Thus the HWR is a poor rectifier.

e. Efficiency(η)

Efficiency of rectifier is a measure of conversion of a.c. input power into useful d.c. output power. It is expressed as the ratio of the d.c. output power to the a.c. input power.

$$\text{i.e., } \eta = P_{dc}/P_{ac} \quad (4.19)$$

where P_{dc} is d.c. power delivered to load and P_{ac} is the ac input power from secondary winding of the transformer.

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{I_{dc}^2 R_L}{I_{ac}^2 (r_d + R_L)} = \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\left(\frac{I_m}{2}\right)^2 (r_d + R_L)} = \frac{0.406 R_L}{(r_d + R_L)} \cong 0.406 = 40.6\%, \quad \text{Since } (r_d \ll R_L)$$

(4.20)

f. Form factor

Form factor is the ratio of rms value to the average value.

$$\text{i.e. } FF = \frac{\text{rms value}}{\text{average value}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} = 1.57 \quad (4.21)$$

g. Peak factor

$$\text{Peak factor is the ratio between peak value and rms value. } PF = \frac{V_m}{(V_m/2)} = 2 \quad (4.22)$$

h. Transformer Utilization Factor (TUF)

TUF is defined as the ratio of d.c. output power to a a.c. power supplied to it by the secondary winding.

$$\text{i.e. } TUF = \frac{P_{dc}}{P_{ac}} \text{ (rated)} \quad (4.23)$$

In case of a single diode HWR, the rated voltage of the secondary winding = $V_m/\sqrt{2}$, but the actual rms value of current flowing through the secondary winding = $I_m/2$ and not $I_m/\sqrt{2}$.

$$TUF = \frac{(I_m/\pi)^2 R_L}{(V_m/\sqrt{2}) (I_m/2)} = \frac{(V_m/\pi)^2 R_L}{\frac{V_m}{\sqrt{2}} \frac{V_m}{2R_L}} = 0.287 \quad (4.24)$$

4.3 FULL WAVE RECTIFIER (FWR)

In FWR, current flows through the load during both half cycles of the input a.c. supply. Like the half wave circuit, a full wave rectifier circuit produces an output voltage or current which is purely DC or has some specified DC component. Full wave rectifiers have some fundamental advantages over their half wave rectifier counterparts. The average (DC) output voltage is higher than for half wave, the output of the full wave rectifier has much less ripple than that of the half wave rectifier producing a smoother output waveform.

There are two types of FWR

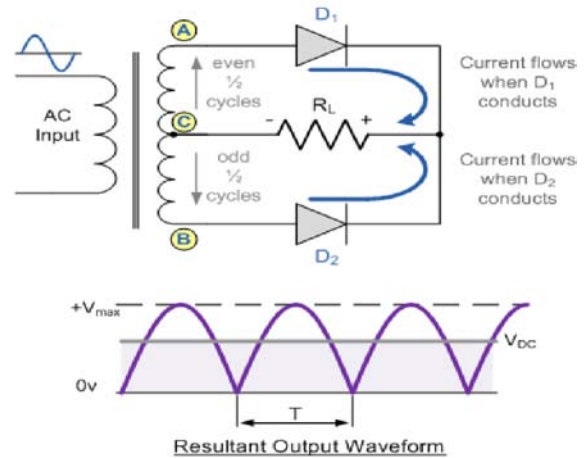
- Centre Tap rectifier
- Bridge Rectifier

4.3.1 Centre Tap Full Wave (CTFW) Rectifier

In a CT **Full Wave Rectifier** circuit two diodes are now used, one for each half of the cycle. A transformer is used whose secondary winding is split equally into two halves with a common centre tapped connection, (C). This configuration results in each diode conducting in turn when its anode terminal is positive with

respect to the transformer centre point C producing an output during both half-cycles, twice that for the half wave rectifier so it is 100% efficient as shown below.

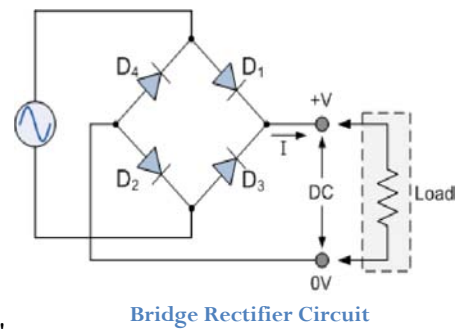
As shown in the circuit the full wave rectifier circuit consists of two diodes connected to a single load resistance (R_L) with each diode taking it in turn to supply current to the load. When point A of the transformer is positive with respect to point B, diode D_1 conducts in the forward direction as indicated by the arrows. When point B is positive (in the negative half of the cycle) with respect to point A, diode D_2 conducts in the forward direction and the current flowing through resistor R is in the same direction for both circuits. As the output voltage across the resistor R is the phasor sum of the two waveforms combined, this type of full wave rectifier circuit is also known as a "bi-phase" circuit.



4.3.2 (Full Wave) Bridge rectifier

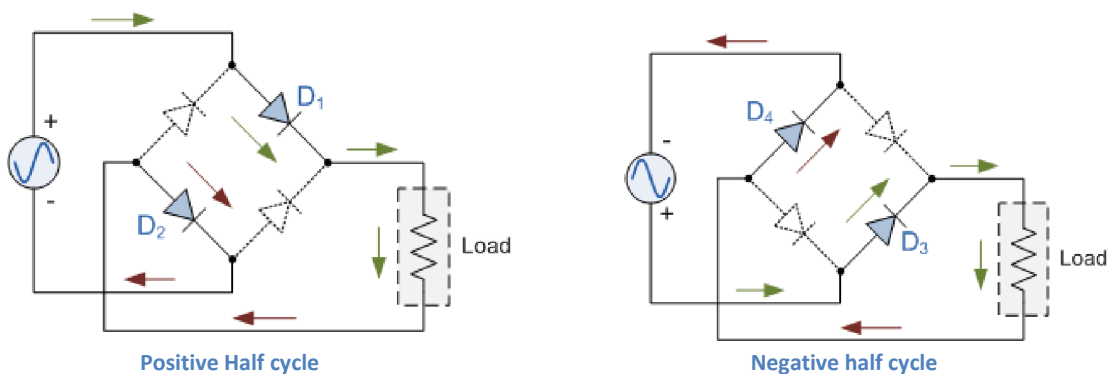
Another type of circuit that produces the same output waveform as the full wave rectifier circuit above is that of the **Full Wave Bridge Rectifier**.

This type of single phase rectifier uses four individual rectifying diodes connected in a closed loop "bridge" configuration to produce the desired output. The main advantage of this bridge circuit is that it does *not require a special centre tapped transformer*, thereby reducing its size and cost. The single secondary winding is connected to one side of the diode bridge network and the load to the other side as shown below.



The four diodes labeled D_1 to D_4 are arranged in "series pairs" with only two diodes conducting current during each half cycle. During the positive half cycle of the supply, diodes D_1 and D_2 conduct in series while diodes D_3 and D_4 are reverse biased and the current flows through the load as shown below.

During the negative half cycle of the supply, diodes D_3 and D_4 conduct in series, but diodes D_1 and D_2 switch off as they are now reverse biased. The current flowing through the load is the same direction as before.



As the current flowing through the load is unidirectional, so the voltage developed across the load is also unidirectional the same as for the previous two diode full-wave rectifier

4.3.3 Performance of full wave rectifier

a. Peak inverse Voltage

In the case of centre tapped full wave rectifier, $PIV = V_m + V_m = 2V_m$. (where the first V_m is the maximum voltage across the load when one diode conducts which must appear at the cathode of the other diode, and the second V_m is the maximum reverse voltage appear at the anode of second (OFF) diode. Hence the peak inverse voltage across the second (OFF) diode in the positive half cycle $= 2V_m$.

In the case of centre tapped full wave rectifier, $PIV = V_m$.

b. D.C output

Mathematically the output wave form of FWR can be described as,

$$i_L = \begin{cases} I_m \sin \omega t & : 0 \leq \omega t \leq \pi \\ I_m \sin \omega t & : \pi \leq \omega t \leq 2\pi \end{cases} \quad (4.25)$$


$$\begin{aligned} \text{Here area} &= \int_0^{2\pi} i_L d(\omega t) \\ &= \int_0^{2\pi} I_m \sin \omega t d(\omega t) = \int_0^{\pi} I_m \sin \omega t d(\omega t) + \int_{\pi}^{2\pi} I_m \sin \omega t d(\omega t) \\ &= \int_0^{\pi} I_m \sin \omega t d(\omega t) \\ I_m [-\cos \omega t]_0^{\pi} &= 2I_m \quad \mathbf{4I_m} \end{aligned} \quad (4.26)$$

$$\text{Time period} = 2\pi \quad (4.27)$$

$$\text{Now, } I_{dc} = I_{av} = \frac{4I_m}{2\pi} = \frac{2I_m}{\pi} \quad (4.28)$$

$$\text{Voltage across } R_L, V_{dc} = I_{dc} R_L = \frac{2I_m R_L}{\pi} \quad (4.29)$$

$$\text{While considering the forward resistance } r_d, I_m = \frac{V_m}{R_L + r_d} \quad (4.30)$$

$$\therefore V_{dc} = \frac{2}{\pi} \frac{V_m}{R_L + r_d} \quad L = \frac{2}{\pi} \frac{V_m}{(1 + \frac{r_d}{R_L})} \cong \frac{2V_m}{\pi} \text{ since } r_d \ll R_L. \quad (4.31)$$

c. R.M.S. Value

The R.M.S. value is the effective value of the current flowing through the load and is given by

$$I_{rms} = \sqrt{\frac{\text{Area under the squared rectified wave over a cycle}}{\text{Time period}}} \quad (4.32)$$

In case of a full wave rectifier,

$$i_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_L^2 d(\omega t)} = i_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \omega t)^2 d(\omega t)} \quad (4.33)$$

$$\begin{aligned} &= \sqrt{\frac{1}{2\pi} \int_0^{\pi} (I_m \sin \omega t)^2 d(\omega t) + \frac{1}{2\pi} \int_{\pi}^{2\pi} (I_m \sin \omega t)^2 d(\omega t)} = \sqrt{\frac{2}{2\pi} \int_0^{\pi} (I_m \sin \omega t)^2 d(\omega t)} \\ &= \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2(\omega t) d(\omega t)} = \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \left\{ \frac{(1 - \cos 2\omega t)}{2} \right\} d(\omega t)} = \sqrt{\frac{I_m^2}{2\pi} \left| \omega t - \frac{\sin \omega t}{2} \right|_0^{\pi}} \\ &= \frac{I_m}{\sqrt{2}} \end{aligned} \quad (4.33)$$

$$\text{Also } V_{rms} = V_m / \sqrt{2} \quad (4.34)$$

This is the rms value of the total load current which include d.c. value and a.c. components. In the out put of rectifier, the instantaneous value of a.c fluctuation is the difference of the instantaneous total value and the d.c. value.

i.e, the instantaneous a.c. value is given as

$$i' = i_L - I_{dc} \quad (4.35)$$

Here the rms value of the a.c component is given by,

$$\begin{aligned} i_{rms}' &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i_L - I_{dc})^2 d(\omega t)} = i_{rms}' = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i_L^2 - 2i_L I_{dc} + I_{dc}^2) d(\omega t)} \\ &= \sqrt{I_{rms}^2 - I_{dc}^2} \quad (\text{refer eqn: 4.13 and 4.14}) \end{aligned} \quad (4.36)$$

d. Ripple factor(γ)

The ripple factor of a FWR is expressed as,

$$\text{i.e.} \quad \gamma = \frac{\text{Ripple Voltage}}{\text{D.C.Voltage}} \quad (4.37)$$

$$\begin{aligned} &= \frac{\text{rms value of a.c.component}}{\text{Average or d.c.component}} \\ &= \frac{V'_{rms}}{V_{dc}} \end{aligned} \quad (4.38)$$

$$\text{or,} \quad \gamma = \frac{I'_{rms}}{I_{dc}} \quad (4.39)$$

$$= \frac{\sqrt{I_{rms}^2 - I_{dc}^2}}{I_{dc}} = \sqrt{\frac{(I_m/\sqrt{2})^2}{(2I_m/\pi)^2} - 1} = 0.482 \quad (4.40)$$

It shows that in case of a FWR output d.c. components exceeds the a.c component. The full wave rectifier is good in rectification process.

e. Efficiency(η)

$$\eta = P_{dc}/P_{ac} \quad (4.41)$$

where P_{dc} is d.c. power delivered to load and P_{ac} is the ac input power from secondary winding of the transformer.

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{\left(\frac{2I_m}{\pi}\right)^2 R_L}{\left(\frac{I_m}{\sqrt{2}}\right)^2 (r_d + R_L)} = \frac{0.812 R_L}{(r_d + R_L)} \cong 0.812 = 81.2\%, \quad \text{Since } (r_d \ll R_L) \quad (4.42)$$

f. Form factor

Form factor is the ratio of rms value to the average value.

$$\text{i.e.} \quad FF = \frac{\text{rms value}}{\text{average value}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11 \quad (4.43)$$

g. Peak factor

$$\text{Peak factor is the ratio between peak value and rms value. } PF = \frac{V_m}{(V_m/\sqrt{2})} = \sqrt{2} \quad (4.44)$$

h. Transformer Utilization Factor (TUF)

TUF is defined as the ratio of d.c. output power to a a.c. power supplied to it by the secondary winding.

$$\text{i.e.} \quad TUF = \frac{P_{dc}}{P_{ac}} \text{ (rated)} \quad (4.45)$$

In case of a Bridge FWR, the rated voltage of the secondary winding = $V_m/\sqrt{2}$ and rms value of current flowing through the secondary winding = $I_m/\sqrt{2}$

$$TUF = \frac{(2I_m/\pi)^2 R_L}{(V_m/\sqrt{2})^2 (I_m/\sqrt{2})} = \frac{(2V_m/\pi)^2}{\frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2} R_L}} = 0.812 \quad (4.46)$$

In case of a CT FWR, $TUF_{\text{primary}} = 0.812$, and $TUF_{\text{secondary}} = 2 \times (TUF \text{ of HWR})$. Hence TUF for a CT rectifier can be expressed as, $TUF = (TUF_{\text{primary}} + TUF_{\text{secondary}})/2 = 0.812 + 287 = 0.693$. ■

Advantages of bridge rectifier

- The peak inverse voltage (PIV) across each diode is v_m and not $2v_m$ as in the case of FWR. Hence the voltage rating of the diodes can be less.
- Centre tapped transformer is not required.
- There is no D.C. current flowing through the transformer since there is no centre tapping and the return path is to the ground. So the transformer utilization factor is high.

Disadvantages

- Four diodes are to be used.
- There is some voltage drop across each diode and so output voltage will be slightly less compared to fwr. But these factors are minor compared to the advantages.

4.3.3 Comparison of Rectifiers

Type	HW	CT FW	FW BR
No. of diodes used	1	2	4
Need of transformer	not necessary	necessary	not necessary
DC current and DC Voltage	$I_m/\pi, & V_m/\pi$	$2I_m/\pi, & 2V_m/\pi$	$2I_m/\pi, & 2V_m/\pi$
RMS current and RMS voltage	$I_m/2 & V_m/2$	$I_m/\sqrt{2}, & V_m/\sqrt{2}$	$I_m/\sqrt{2}, & V_m/\sqrt{2}$
Ripple factor(γ)	1.21	0.48	0.48
Efficiency(η)	40.6%	81.2%	81.2%
PIV	V_m	$2V_m$	V_m
TUF	0.287	0.812	0.693
Form factor	1.57	1.11	1.11
Peak factor	2	$\sqrt{2}$	$\sqrt{2}$
Ripple frequency	f	2f	2f